

# Product-Mix Auctions: Examples, and Introduction to the PMA web app

## Companion Slide Set

*The code is being developed further; in particular, some graphs may be differently titled on the screenshots than in the current version of the web app.*

Please send any questions to [paul.klemperer@nuffield.ox.ac.uk](mailto:paul.klemperer@nuffield.ox.ac.uk)

### Product-Mix Auctions

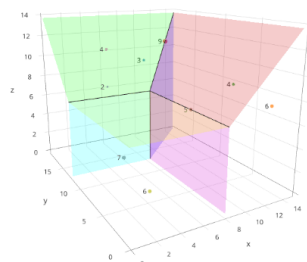
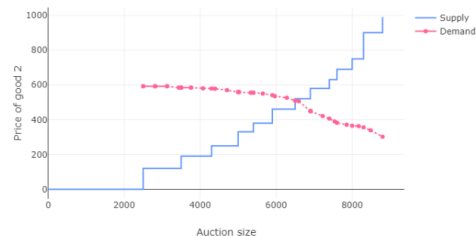
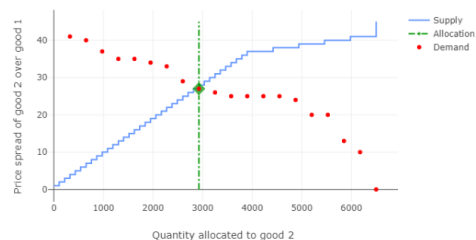
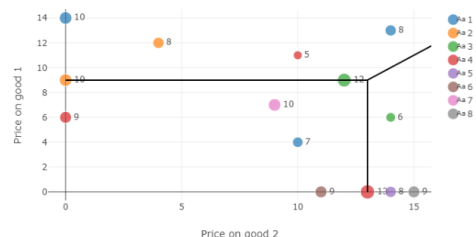
We provide open-source software for three different Product-Mix Auctions. (All Product-Mix Auctions are single-round sealed-bid auctions for multiple units of multiple distinct goods.)

- **Standard Version.**
  - This is a generalisation of the design that is the basis of the "ILTR" auction that is now regularly used by the Bank of England — see [paper](#).
  - It permits flexible buyer and seller preferences over an arbitrary number of goods.
- **Version for Budget-constrained bidders.**
  - This version was originally designed for the Government of Iceland.
  - As in the standard version, bidders simultaneously make sets of bids. In this case, however, each of a bidder's bids can specify a total budget to be spent.
- **"Positive and negative dot-bids" Version.**
  - This version extends the standard version to permit all participants (buyers and seller) to express *any* strong substitute preferences for indivisible goods.

This software is available both as a command-line program `pma`, and a web application `pma-server` (all open-source). The latter provides a single-user interface allowing an auction specification to be constructed in a web browser form. We host the web application here and also provide the code so users can host it locally.

Please contact [Elizabeth Baldwin](#) or [Paul Klemperer](#) for the password to access this software, or for further information about it; its use is free of charge.

[Download software and documentation \(requires password\)](#)



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### 7.1 not in presentation

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Section 1

Section 1.1

# 1. Independent Auctions

## 1.1 Auctioning a Single Variety

Auction input

Supply

Supply Ordering  
Horizontal/Supply

Supply curves for each good

Supply curve for good 1

Units	Price
30	0

Bidders

Bidder 1   Bidder

Bid	Units	Price	
1	4	17	<input type="button" value="x"/>
2	8	11	<input type="button" value="x"/>
3	5	7	<input type="button" value="x"/>

Bidder 2   Bidder

Bid	Units	Price
1	11	9

Bidder 3   Bidder

Bid	Units	Price
1	11	8

Bidder 4   Bidder

Bid	Units	Price
1	11	6

Auction output

Prices and total allocations

	Good 1
Price	8
Allocation	30

Allocations

Bidder	Good 1
1	12
2	11
3	7

Bid allocations

Allocation of bids to goods



Supply curve and unadjusted demand\* for good 1



The bottom-right corner indicates the example corresponding to the current slide —→ demoSingle

# 1. Independent Auctions

## 1.1 Auctioning a Single Variety

Supply  
curves

Bidders/  
Bids

Auction input

Supply

Supply Ordering  
Horizontal/Supply

Supply curves for each good

Supply curve for good 1

Units	Price
30	0

Bidders

Bidder 1   Bidder

Bid	Units	Price	
1	4	17	<input type="button" value="x"/>
2	8	11	<input type="button" value="x"/>
3	5	7	<input type="button" value="x"/>

Bidder 2   Bidder

Bid	Units	Price
1	11	9

Bidder 3   Bidder

Bid	Units	Price
1	11	8

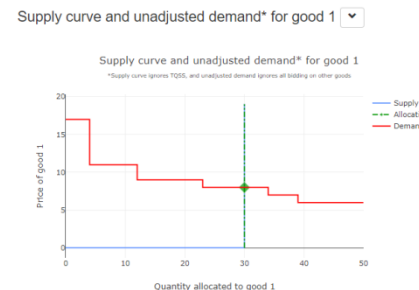
Bidder 4   Bidder

Bid	Units	Price
1	11	6



Prices &  
Allocations

Allocations  
of bids to  
goods



Demand,  
supply &  
allocations

# 1.1 Auctioning a Single Variety Bidders/Bids

Bidder 1 ▼ ✕ Bidder

Bid	Units	Price	
1	<input type="text" value="4"/> 4	<input type="text" value="17"/> 17	<span>✕</span>
2	<input type="text" value="8"/> 8	<input type="text" value="11"/> 11	<span>✕</span>
3	<input type="text" value="5"/> 5	<input type="text" value="7"/> 7	<span>✕</span>

+ Bid

Bidder 2 ▼ ✕ Bidder

Bid	Units	Price
1	<input type="text" value="11"/> 11	<input type="text" value="9"/> 9

+ Bid

Bidder 3 ▼ ✕ Bidder

Bid	Units	Price
1	<input type="text" value="11"/> 11	<input type="text" value="8"/> 8

+ Bid

Bidder 4 ▼ ✕ Bidder

Bid	Units	Price
1	<input type="text" value="11"/> 11	<input type="text" value="6"/> 6

+ Bid

# 1.1 Auctioning a Single Variety Supply Curves

Supply

Supply Ordering

HorizontalSupply

Supply curves for each good

Supply curve for good 1

Units	Price
<input type="text" value="30"/> 30	<input type="text" value="0"/> 0

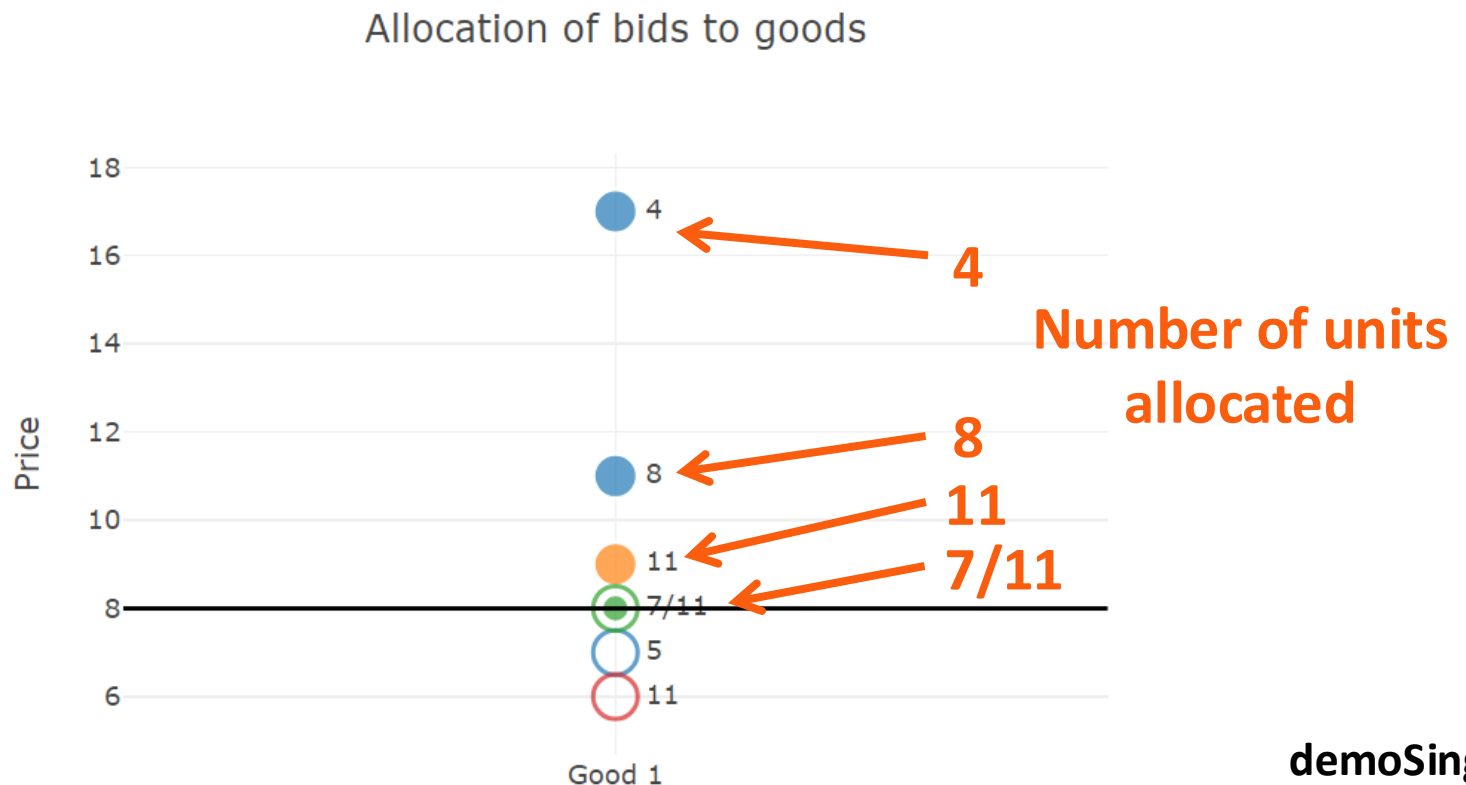
+ Supply step

+ Supply curve for good

# 1.1 Auctioning a Single Variety

## Allocation of Bids to Goods

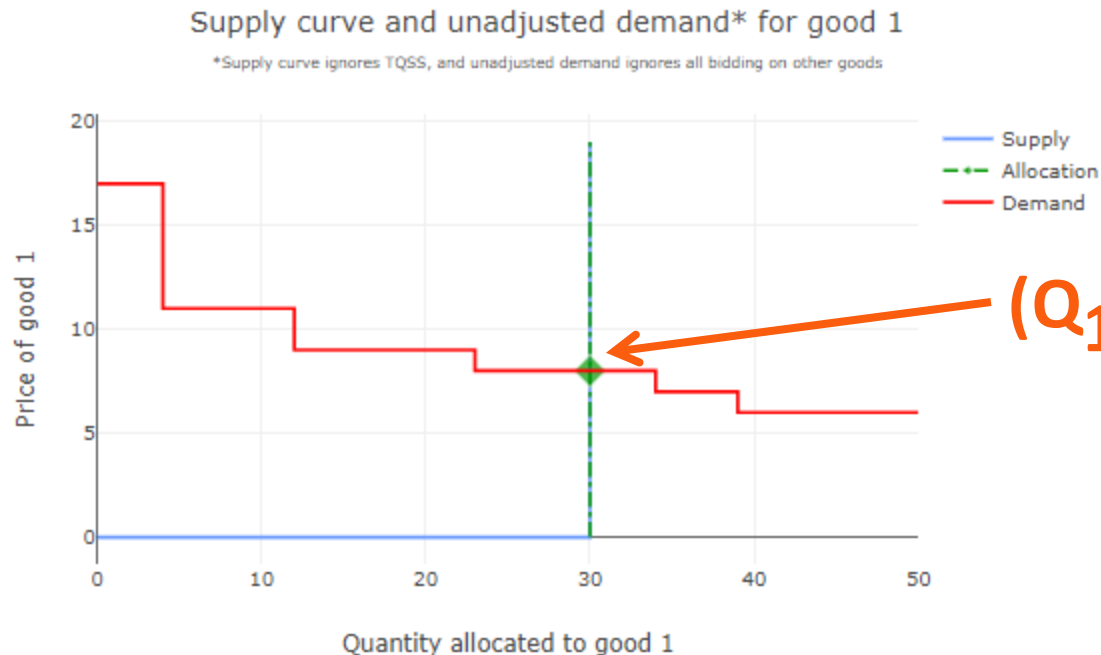
(auctioneer is selling 30 units; reserve price 0)





# 1.1 Auctioning a Single Variety Demand, Supply, & Allocation

Supply curve and unadjusted demand\* for good 1



# 1.1 Auctioning a Single Variety Bidders/Bids

Bidder 1 ▼ ✕ Bidder

Bid	Units	Price	
1	11 11	17	✕
2	11 11	11	✕
3	11 11	7	✕

+ Bid

Bidder 2 ▼ ✕ Bidder

Bid	Units	Price
1	11	9

+ Bid

Bidder 3 ▼ ✕ Bidder

Bid	Units	Price
1	11	8

+ Bid

Bidder 4 ▼ ✕ Bidder

Bid	Units	Price
1	11	6

+ Bid

# 1.1 Auctioning a Single Variety Prices & Allocation of Bids to Goods

Auction output

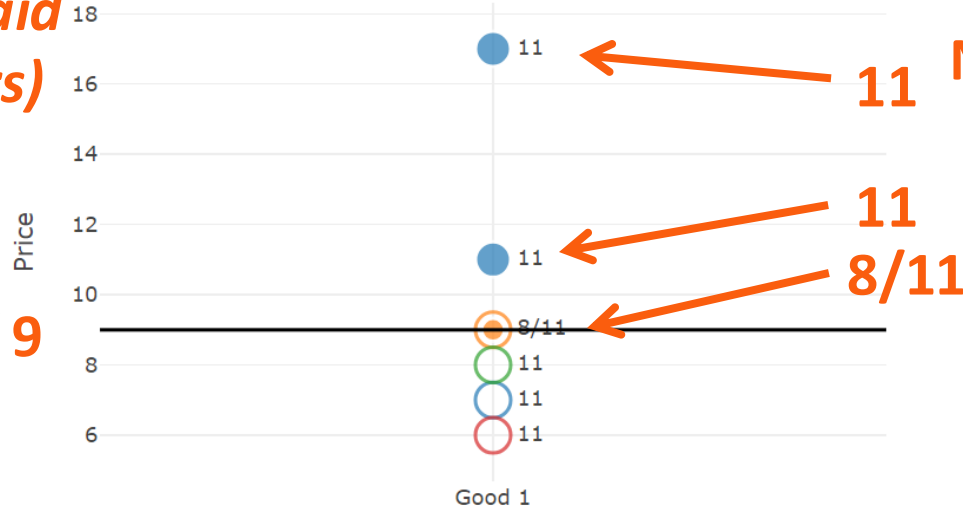
Prices and total allocations

	Good 1
Price	9
Allocation	30

Allocations

Bidder	Good 1
1	22
2	8

**Uniform Price**  
*(Same Price paid  
by All Winners)*



**Number of units  
allocated**

## Section 1.2

# 1. Independent Auctions

## 1.2 Two Separate (Uniform-Price) Auctions

Supply  
curves

Bidders/  
Bids

Auction input

Supply

Supply Ordering  
HorizontalSupply

Supply curves for each good

Supply curve for good 1  Supply curve

Units	Price
30	0

Supply step

Supply curve for good 2  Supply curve

Units	Price
30	0

Supply step

Supply curve

Bidders

Bidder 1  Bidder

Bid	Units	Price	Price
1	11	17	0
2	11	11	0
3	11	7	0
4	11	0	15

Bid

Bidder 2  Bidder

Bid	Units	Price	Price
1	11	9	0
2	11	0	10

Bid

Bidder 3  Bidder

Bid	Units	Price	Price
1	11	8	0

Bid

Bidder 4  Bidder

Bid	Units	Price	Price
1	11	6	0
2	11	0	13

Bid

Bidder 5  Bidder

Bid	Units	Price	Price
1	11	8	14

Auction output

Prices and total allocations

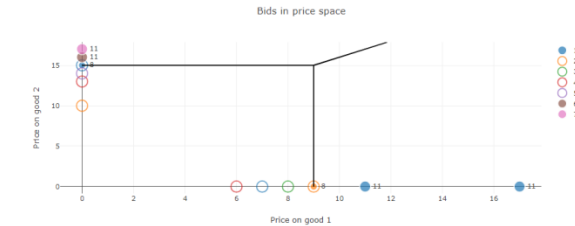
	Good 1	Good 2
Price	9	15
Allocation	30	30

Allocations

Bidder	Good 1	Good 2
1	22	8
2	8	0
6	0	11
7	0	11

Bid allocations

Bids in price space



Prices &  
Allocations

Bids in  
price space

Demand,  
supply &  
allocations

# 1.2 Two Separate (Uniform-Price) Auctions

## Supply Curves

Supply

Supply Ordering

HorizontalSupply

Supply curves for each good

Supply curve for good 1

Units	Price
<input type="text" value="30"/> 30	<input type="text" value="0"/> 0

Supply curve for good 2

Units	Price
<input type="text" value="30"/> 30	<input type="text" value="0"/> 0

# 1.2 Two Separate (Uniform-Price) Auctions

## Bidders/Bids

Quantity bid for      Good 1 Price bid      Good 2 Price bid

Bidders   Bidder

Bidder 1  Bidder

Bid	Units	Price	Price	
1	11	17	0	<input type="button" value="x"/>
2	11	11	0	<input type="button" value="x"/>
3	11	7	0	<input type="button" value="x"/>
4	11	0	15	<input type="button" value="x"/>

Bidder 2  Bidder

Bid	Units	Price	Price	
1	11	9	0	<input type="button" value="x"/>
2	11	0	10	<input type="button" value="x"/>

Bidder 3  Bidder

Bid	Units	Price	Price	
1	11	8	0	

Quantity bid for      Good 1 Price bid      Good 2 Price bid

Bidders   Bidder

Bidder 5  Bidder

Bid	Units	Price	Price	
1	11	6	0	<input type="button" value="x"/>
2	11	0	13	<input type="button" value="x"/>

Bidder 6  Bidder

Bid	Units	Price	Price	
1	11	0	14	

Bidder 7  Bidder

Bid	Units	Price	Price	
1	11	0	17	

demoTwoSeparate

# 1.2 Two Separate (Uniform-Price) Auctions

## Prices & Allocations

Auction output

Prices and total allocations

	Good 1		Good 2	
Price	<input type="text" value="9"/>	9	<input type="text" value="15"/>	15
Allocation	<input type="text" value="30"/>	30	<input type="text" value="30"/>	30

Allocations

Bidder	Good 1		Good 2	
1	<input type="text" value="22"/>	22	<input type="text" value="8"/>	8
2	<input type="text" value="8"/>		<input type="text" value="0"/>	
6	<input type="text" value="0"/>		<input type="text" value="11"/>	
7	<input type="text" value="0"/>		<input type="text" value="11"/>	



# 1.2 Two Separate (Uniform-Price) Auctions

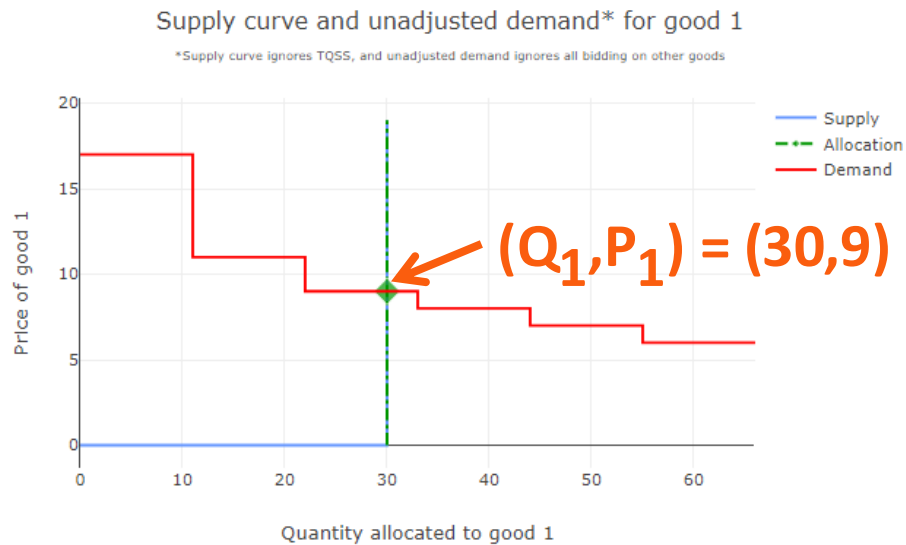
## Bids in price space & Allocations of bids to goods



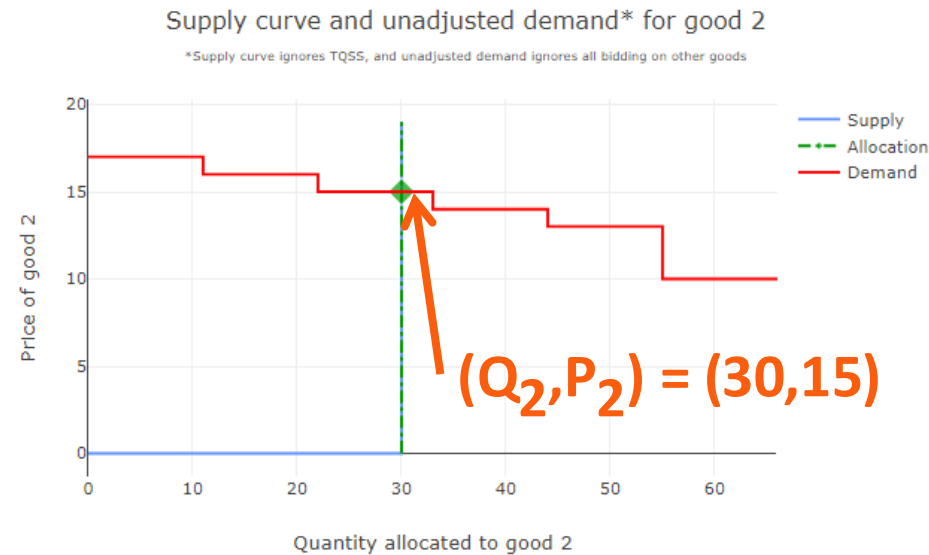
# 1.2 Two Separate (Uniform-Price) Auctions

## Demand, Supply & Allocations

Supply curve and unadjusted demand\* for good 1



Supply curve and unadjusted demand\* for good 2



Section 2

Section 2.1

## 2. Combining the Auctions

### 2.1 Introducing the Total Quantity Supply Schedule (TQSS)

Supply curves for each good

Supply curve for good 1

Units	Price
<input type="text" value="60"/>	<input type="text" value="0"/>

Supply curve for good 2

Units	Price
<input type="text" value="60"/>	<input type="text" value="0"/>

TQSS

TQSS enabled

Price normalisation and scaling

Normalised - constrain supply (may constrain quantities below supply cu

TQSS Steps

Units	Price
<input type="text" value="60"/>	<input type="text" value="0"/>

60 [was 30]

60 [was 30]

Enabled [was Disabled]

TQSS = Total Quantity Supply Schedule  
(Here willing to sell up to 60 of good 1,  
and willing to sell up to 60 of good 2,  
but no more than 60 in total.)

# 2.1 Introducing the Total Quantity Supply Schedule (TQSS)

## Demand, Supply & Allocations

Supply curves for each good

Supply curve for good 1  Supply curve

Units	Price
60	0

60 [was 30]

Supply curve for good 2  Supply curve

Units	Price
60	0

60 [was 30]

TQSS

TQSS enabled

Enabled

Enabled [was Disabled]

Price normalisation and scaling

Normalised - constrain supply (may constrain quantities below supply cu

TQSS Steps

Units	Price
60	0

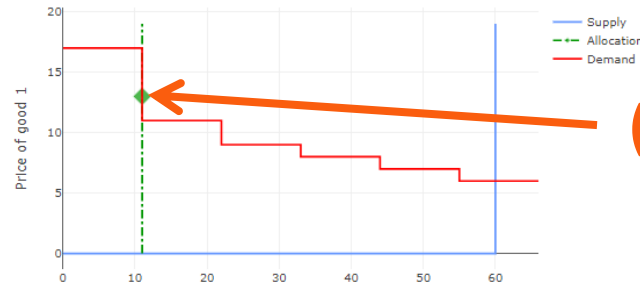
60

0

**"total demand" (in bottom graph) = horizontal sum of demands (in top two graphs)—see next slide**

Supply curve and unadjusted demand\* for good 1

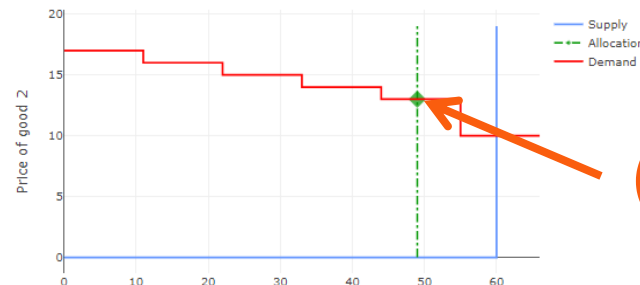
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$(Q_1, P_1) = (11, 13)$

Supply curve and unadjusted demand\* for good 2

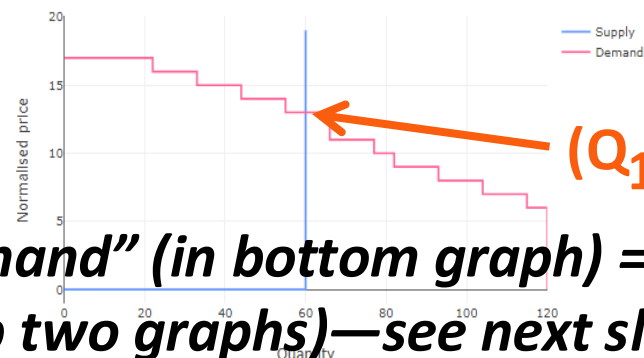
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$(Q_2, P_2) = (49, 13)$

TQSS and (normalised) total demand

When some bids are paired, graph of demand is approximate for quantities exceeding total actually allocated.



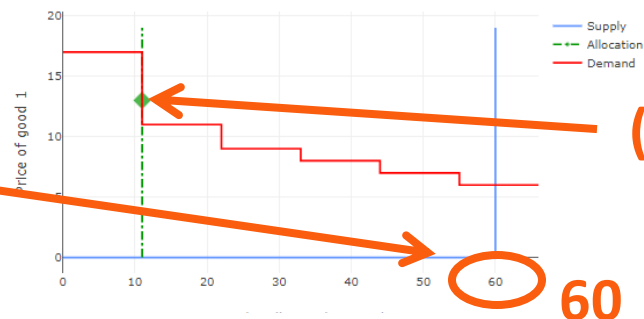
$(Q_1+Q_2, P) = (60, 13)$

# 2.1 Introducing the Total Quantity Supply Schedule (TQSS) Demand, Supply & Allocations

Auctioneer is willing to sell  
up to **60 of good 1**,

Supply curve and unadjusted demand\* for good 1

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$$(Q_1, P_1) = (11, 13)$$

and up to **60 of good 2**,

Supply curve and unadjusted demand\* for good 2

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods

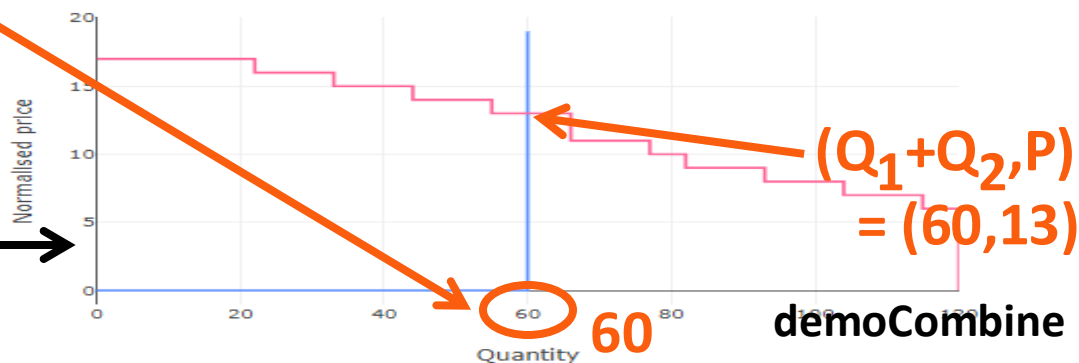


$$(Q_2, P_2) = (49, 13)$$

but no more than **60 in  
total** (Total Quantity  
Supply Schedule)

TQSS and (normalised) total demand

When some bids are paired, graph of demand is approximate for quantities exceeding total actually allocated.



*(This is the previous  
screenshot, rescaled)*

demoCombine

# 2.1 Introducing the Total Quantity Supply Schedule (TQSS)

## Bids in price space & Allocation of bids to goods



## Section 2.2



# 2. Combining the Auctions

## 2.2 Simple Examples

### Supply curves for each good

Supply curve for good 1 ▼ ✕ Supply curve

Units	Price
60	0

Supply curve for good 2 ▼ ✕ Supply curve

Units	Price
60	0

TQSS ▼

TQSS enabled

Enabled

Price normalisation and scaling

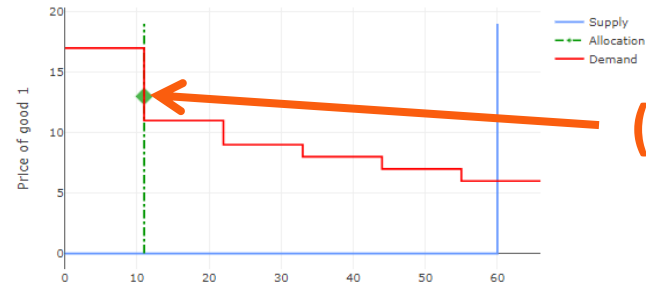
Normalised - constrain supply (may constrain quantities below supply cu

TQSS Steps

Units	Price
60	0

Supply curve and unadjusted demand\* for good 1

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$$(Q_1, P_1) = (11, 13)$$

Supply curve and unadjusted demand\* for good 2

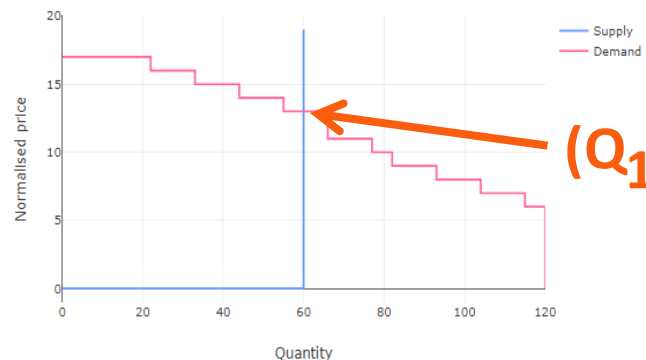
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$$(Q_2, P_2) = (49, 13)$$

TQSS and (normalised) total demand

When some bids are paired, graph of demand is approximate for quantities exceeding total actually allocated.



$$(Q_1 + Q_2, P) = (60, 13)$$

demoCombine

## 2.2 Simple Examples

### Constraining maximum quantity of good

Supply curves for each good

Supply curve for good 1

Units	Price
<input type="text" value="60"/>	<input type="text" value="0"/>

Supply curve for good 2

Units	Price
<input type="text" value="45"/>	<input type="text" value="0"/>

TQSS

TQSS enabled

Enabled

Price normalisation and scaling

Normalised - constrain supply (may constrain quantities below supply cu

TQSS Steps

Units	Price
<input type="text" value="60"/>	<input type="text" value="0"/>

45 [was 60]

## 2.2 Simple Examples – Constraining maximum Q of good 2

### Demand, supply & allocations

Supply curves for each good

Supply curve for good 1  Supply curve

Units	Price
<input type="text" value="60"/>	<input type="text" value="0"/>

Supply curve for good 2  Supply curve

Units	Price
<input type="text" value="45"/>	<input type="text" value="0"/>

45 [was 60]

TQSS

TQSS enabled

Enabled

Price normalisation and scaling

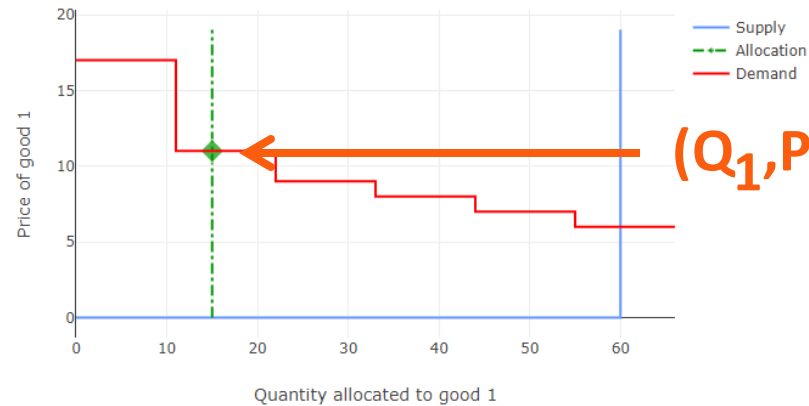
Normalised - constrain supply (may constrain quantities below supply curve)

TQSS Steps

Units	Price
<input type="text" value="60"/>	<input type="text" value="0"/>

Supply curve and unadjusted demand\* for good 1

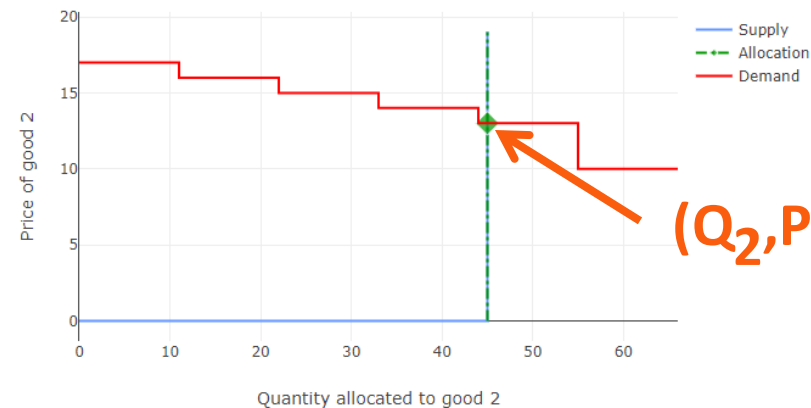
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$(Q_1, P_1) = (15, 11)$

Supply curve and unadjusted demand\* for good 2

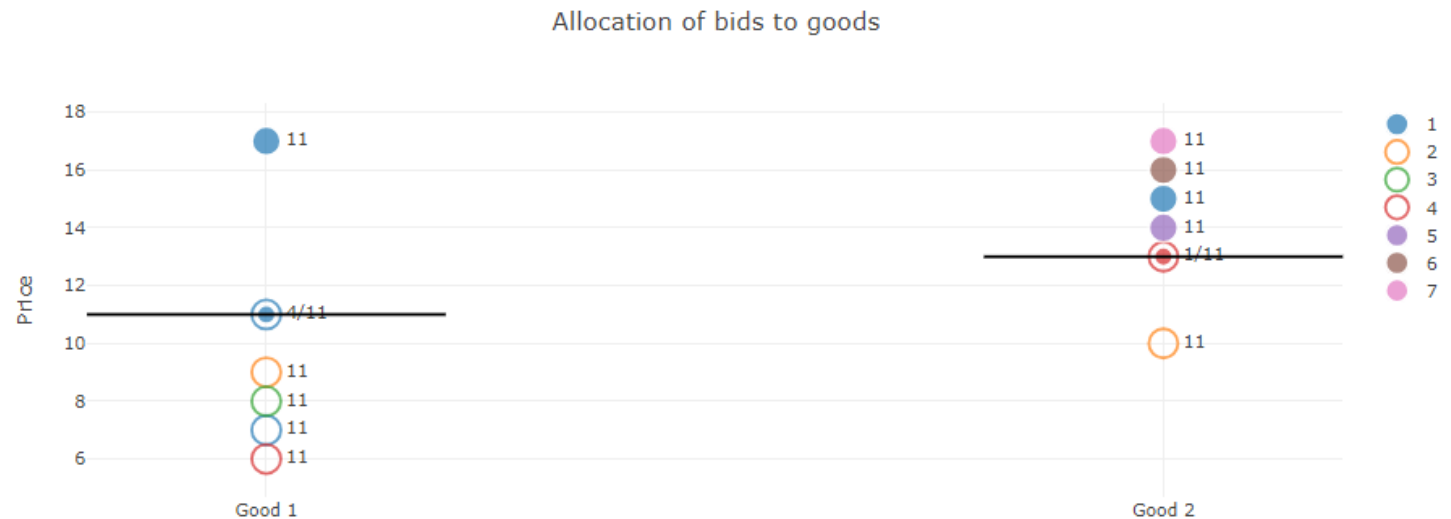
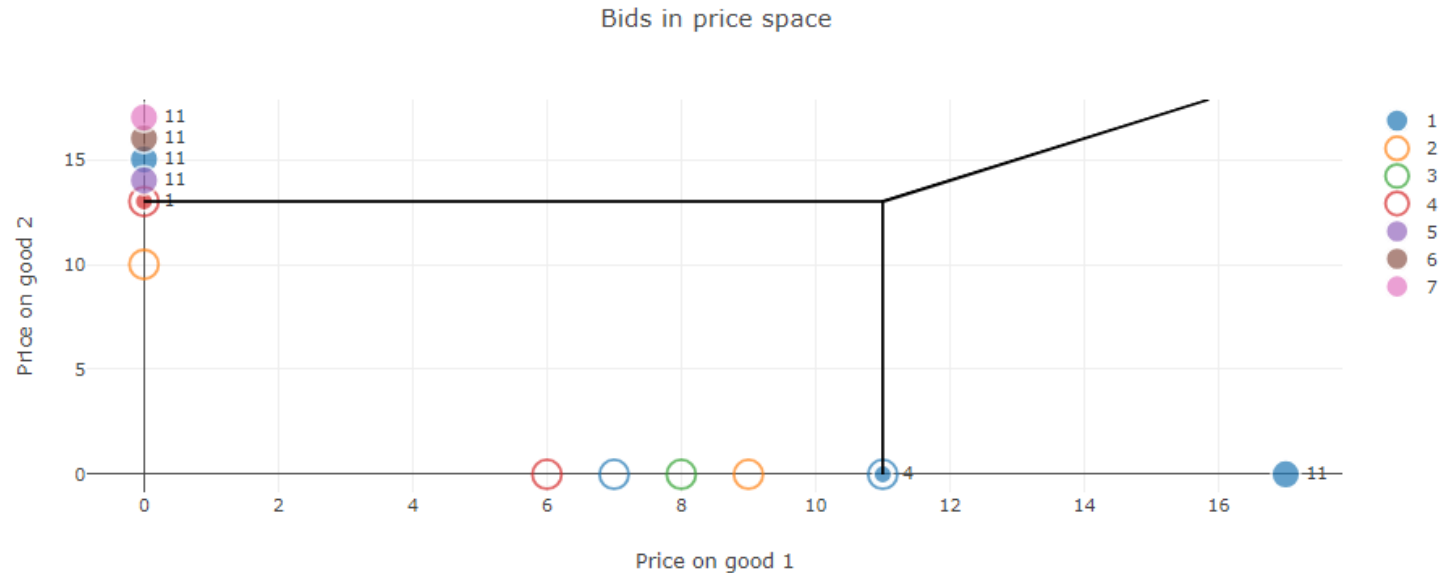
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$(Q_2, P_2) = (45, 13)$

## 2.2 Simple Examples – Constraining maximum Q of good 2

### Bids in price space & Allocations of bids to goods



## 2.2 Simple Examples

### Fixing price difference between goods

-> cf. handicapping good 2

Supply curves for each good

Supply curve for good 1  Supply curve

Units	Price
<input type="text" value="60"/>	<input type="text" value="0"/>

Supply curve for good 2  Supply curve

Units	Price
<input type="text" value="60"/>	<input type="text" value="8"/>

TQSS  TQSS

TQSS enabled

Price normalisation and scaling

TQSS Steps

Units	Price
<input type="text" value="60"/>	<input type="text" value="0"/>

60 [was 45]

8 [was 0]

# 2.2 Simple Examples – Fixing price difference between goods

## Demand, Supply & Allocations

Supply curves for each good

Supply curve for good 1

Units	Price
60	0

Supply curve for good 2

Units	Price
60	8

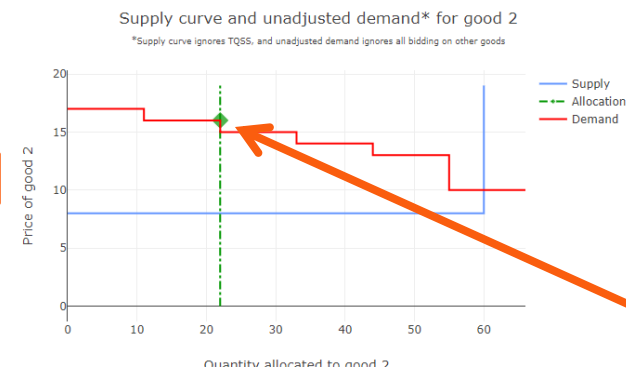
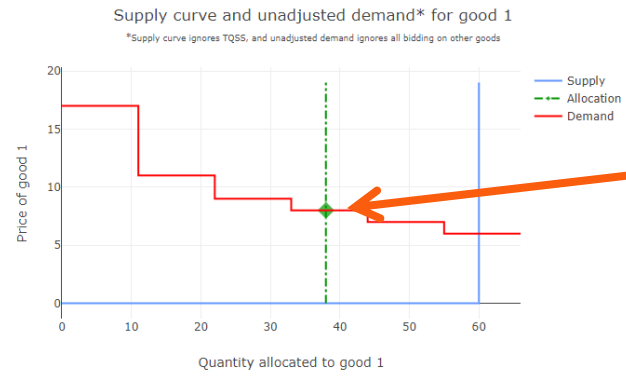
TQSS

TQSS enabled

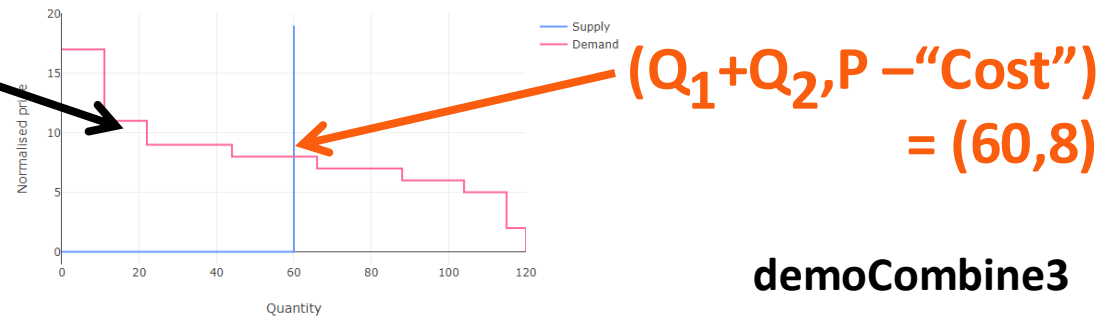
Price normalisation and scaling

TQSS Steps

Units	Price
60	0



**“total demand”= horizontal sum of P –“Cost” curves**



# 2.2 Simple Examples – Fixing price difference between goods

## Demand, Supply & Allocations

Supply curves for each good

Supply curve for good 1

Units	Price
60	0

Supply curve for good 2

Units	Price
60	8

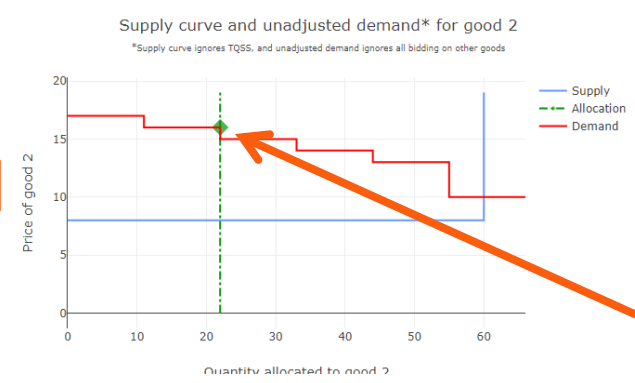
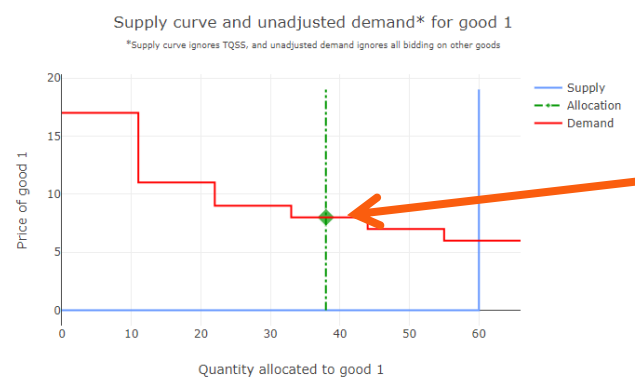
TQSS

TQSS enabled

Price normalisation and scaling

TQSS Steps

Units	Price
60	0

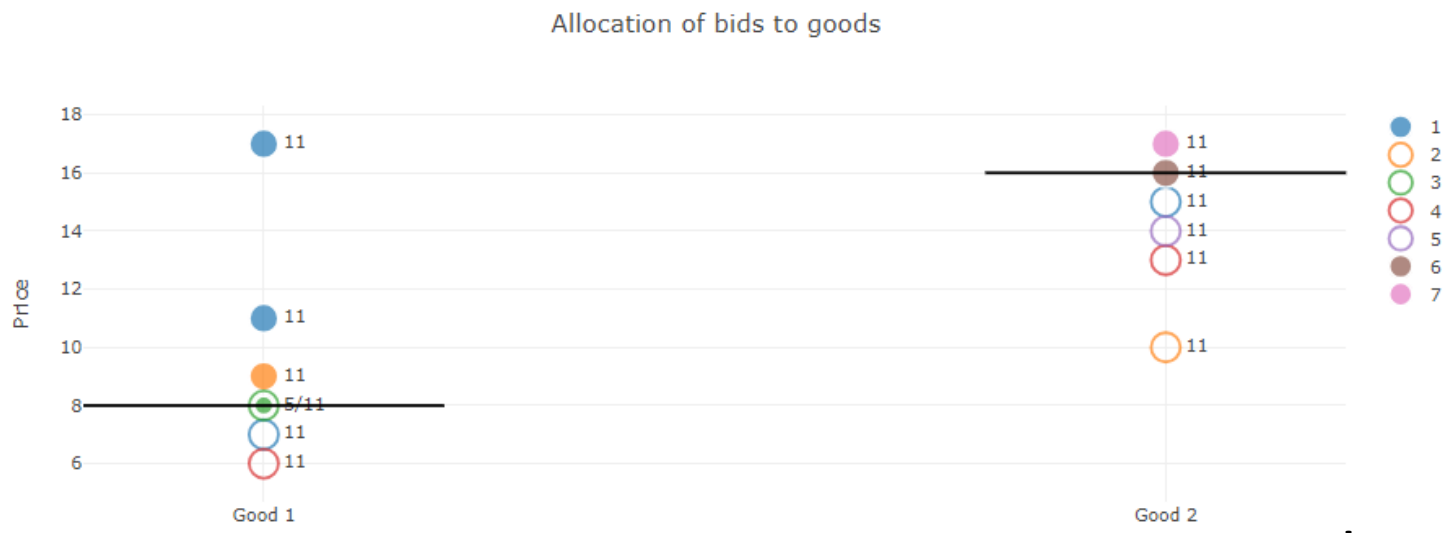


**“total demand”= horizontal sum of P –“Cost” curves (rescaled)**



# 2.2 Simple Examples – Fixing price difference between goods

## Bids in price space & Allocations of bids to goods





## 2.2 Simple Examples

### Upward sloping supply curve for good 2

Supply curve for good 1  Supply curve for good

Units	Price
60	0

Supply curve for good 2  Supply curve for good

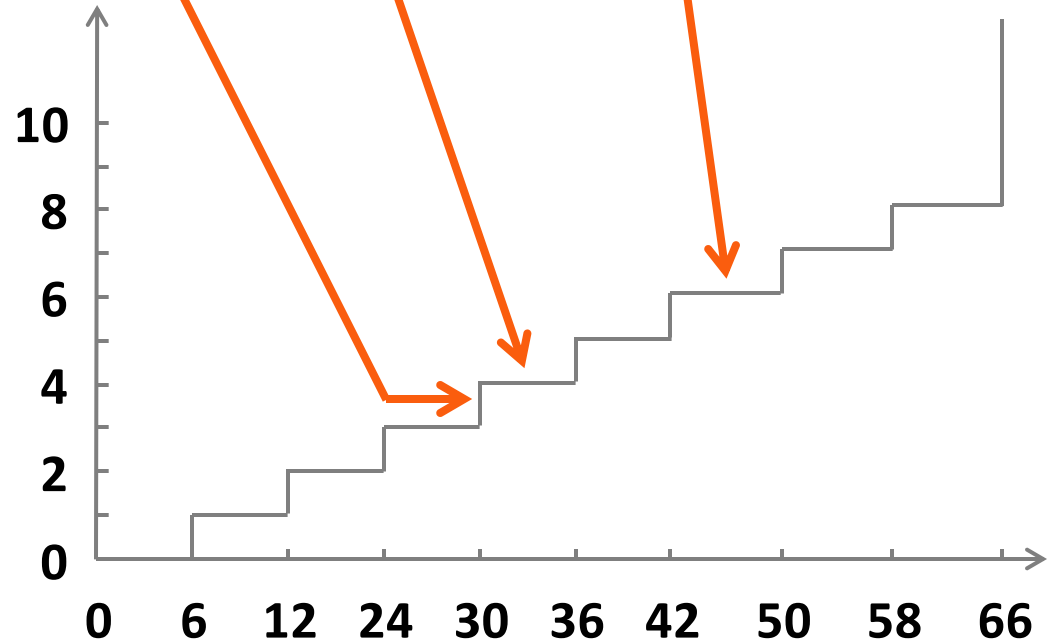
Units	Price	
6	0	×
6	1	×
6	2	×
6	3	×
6	4	×
6	5	×
8	6	×
8	7	×
8	8	×

TQSS Steps

Units	Price
60	0

-> Price difference is increasing in quantity of good 2

Construction of supply curve for good 2:  
steps of length 6 and length 8 and of height 1



demoUpwardSloping

# 2.2 Simple Examples – Upward sloping supply curve for good 2

## Demand, Supply & Allocations

Supply curve for good 1   Supply curve for good

Units	Price
60	0

Supply curve for good 2   Supply curve for good

Units	Price	
6	0	<input checked="" type="button" value="x"/>
6	1	<input checked="" type="button" value="x"/>
6	2	<input checked="" type="button" value="x"/>
6	3	<input checked="" type="button" value="x"/>
6	4	<input checked="" type="button" value="x"/>
6	5	<input checked="" type="button" value="x"/>
8	6	<input checked="" type="button" value="x"/>
8	7	<input checked="" type="button" value="x"/>
8	8	<input checked="" type="button" value="x"/>

TQSS Steps

Units	Price
60	0

Supply curve and unadjusted demand\* for good 1

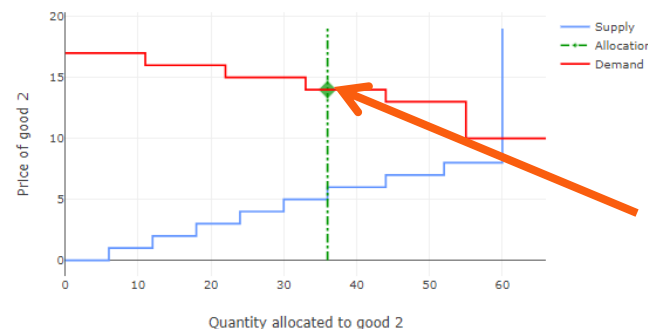
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$$(Q_1, P_1) = (24, 9)$$

Supply curve and unadjusted demand\* for good 2

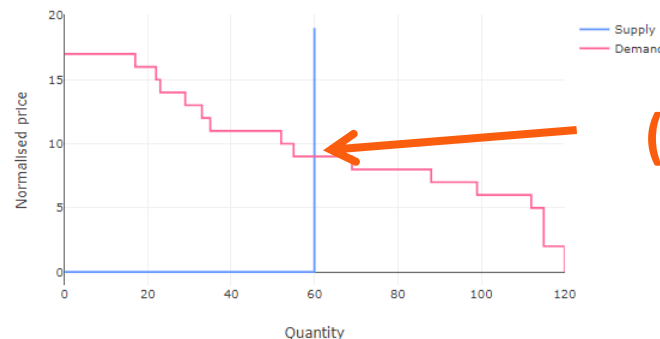
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$$(Q_2, P_2) = (36, 14)$$

TQSS and (normalised) total demand

When some bids are paired, graph of demand is approximate for quantities exceeding total actually allocated.

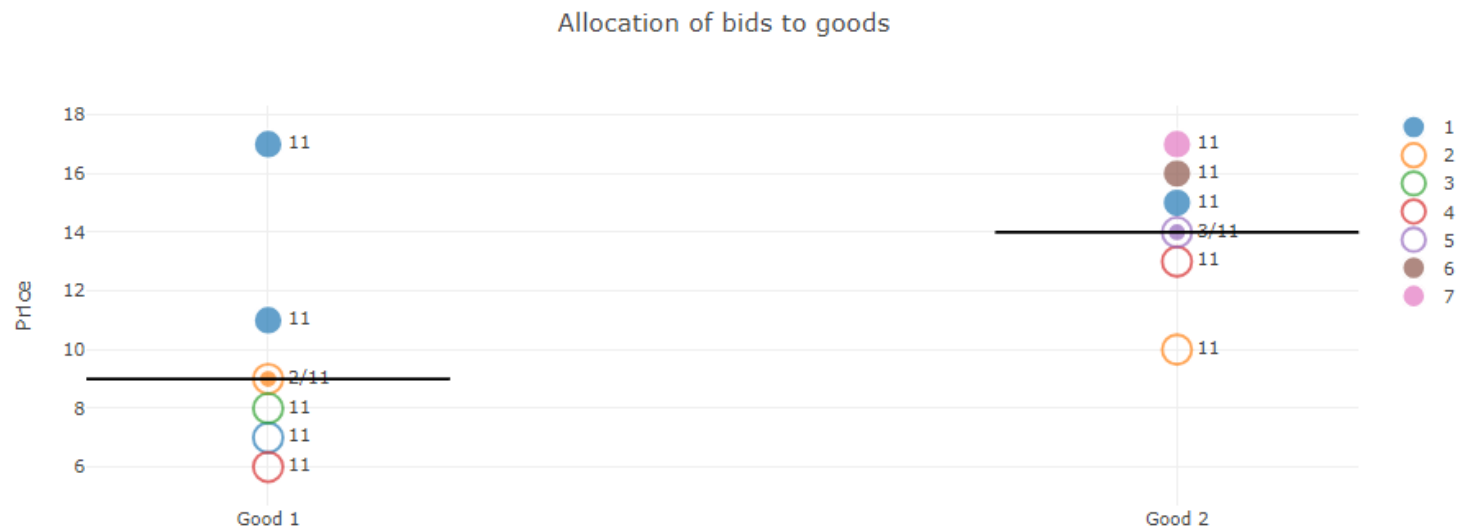


$$(Q_1 + Q_2, P - \text{"Cost"}) = (60, 9)$$

demoUpwardSloping

## 2.2 Simple Examples – Upward sloping supply curve for good 2

### Bids in price space & Allocations of bids to goods



## 2.2 Simple Examples

### Example of a “Paired Bid”

Bidder 1 ▼ ✕ Bidder

Bid	Units	Price	Price	
1	11	17	0	✕
2	11	11	0	✕
3	11	7	0	✕
4	11	0	15	✕
5	2	13	17	✕

+ Bid

Bidder 2 ▼ ✕ Bidder

Bid	Units	Price	Price	
1	11	9	0	✕
2	11	0	10	✕

+ Bid

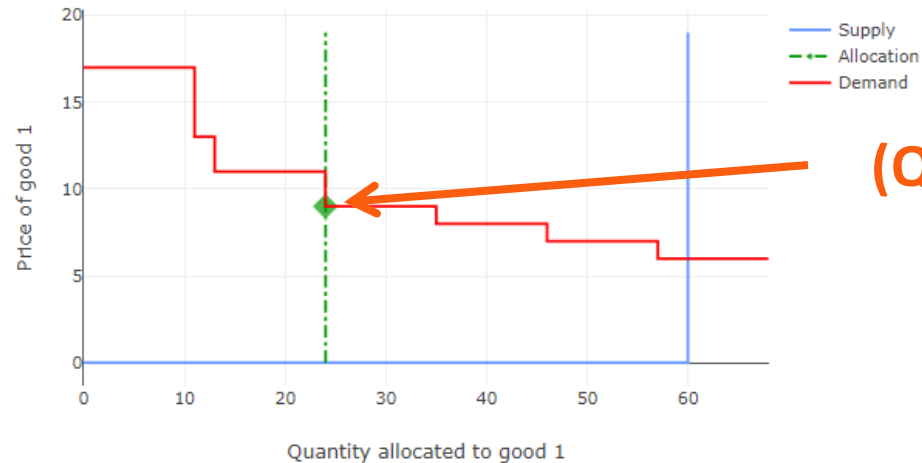
Bidder 3 ▼ ✕ Bidder

Bid	Units	Price	Price
1	11	8	0

## 2.2 Simple Examples – Example of a “Paired Bid” Demand, Supply & Allocations

Supply curve and unadjusted demand\* for good 1

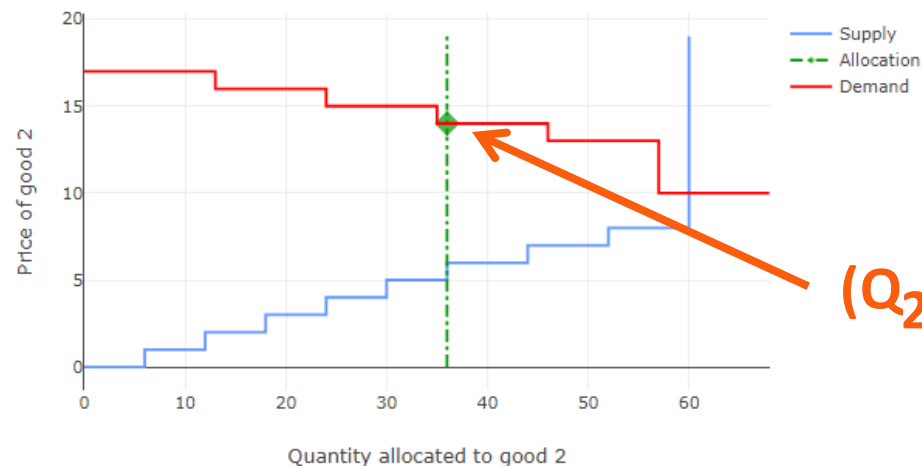
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$$(Q_1, P_1) = (24, 9)$$

Supply curve and unadjusted demand\* for good 2

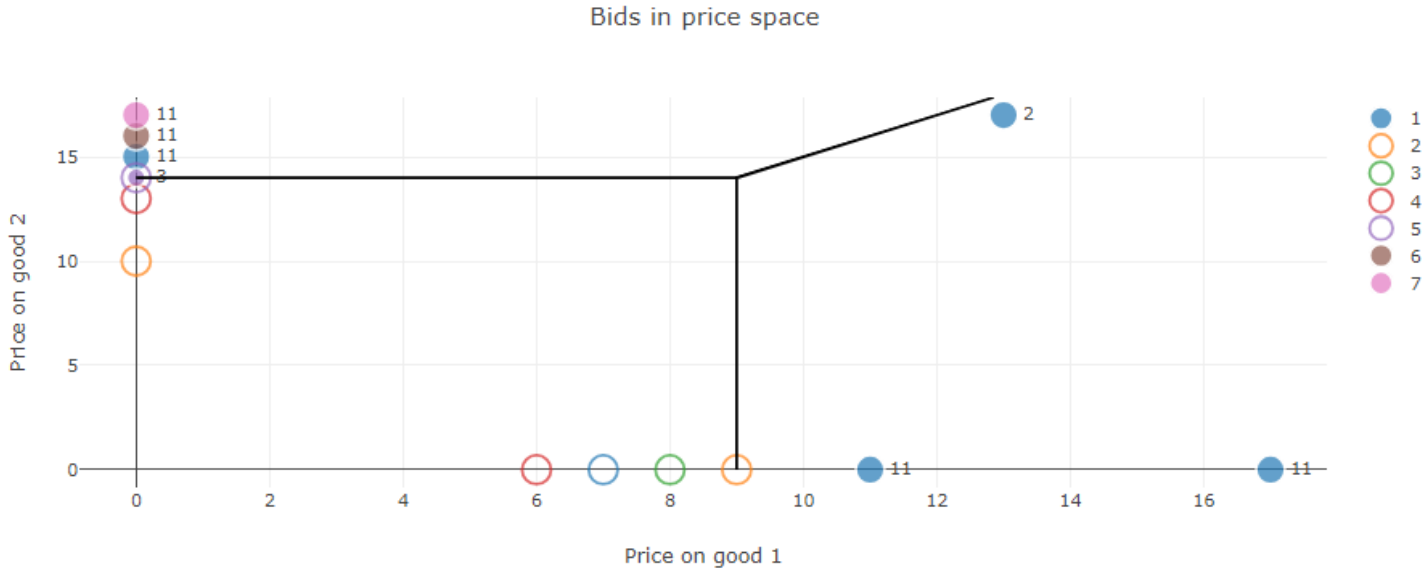
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$$(Q_2, P_2) = (36, 14)$$

## 2.2 Simple Examples – Example of a “Paired Bid”

### Bids in price space & Allocations of bids to goods



## Section 3

### Section 3.1

# 3. The “Vertical” Representation, and the Bank of England’s Auction

## 3.1 Vertical representation

Supply

Supply Ordering

VerticalSupply

“Vertical Supply” [was “Horizontal Supply”]

Supply curves for each good

Supply curve for good 1 and higher   Supply curve

Units	Price
60	0

Supply step

Supply curve for good 2   Supply curve

Units	Price	
6	0	<input type="button" value="x"/>
6	1	<input type="button" value="x"/>
6	2	<input type="button" value="x"/>
6	3	<input type="button" value="x"/>

TQSS

TQSS enabled

Disabled

“Disabled” [was “Enabled”]

Bidder 1   Bidder

Bid	Units	Price	Price	
1	11	17	0	<input type="button" value="x"/>
2	11	11	0	<input type="button" value="x"/>
3	11	7	0	<input type="button" value="x"/>
4	11	0	15	<input type="button" value="x"/>
5	2	13	17	<input type="button" value="x"/>

Bid

Bidder 2   Bidder

Bid	Units	Price	Price	
1	11	9	0	<input type="button" value="x"/>
2	11	0	10	<input type="button" value="x"/>

Bid

Bidder 3   Bidder

Bid	Units	Price	Price
1	11	8	0

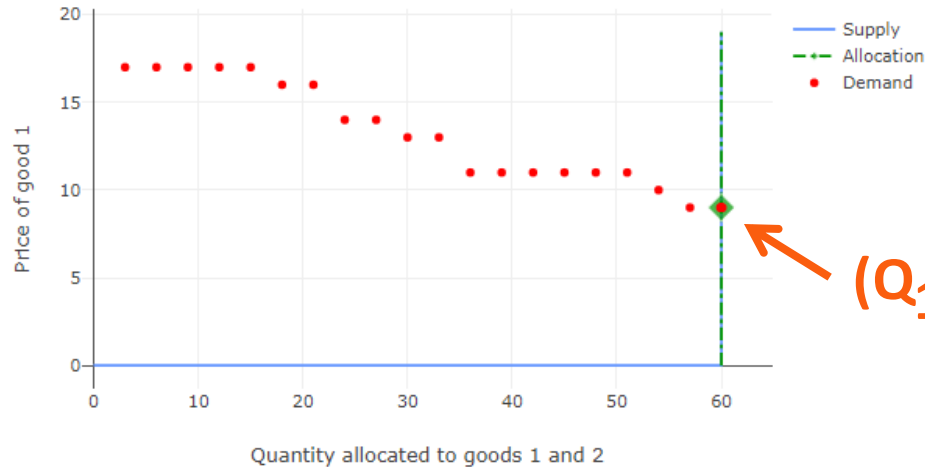
demoVertical



# 3.1 Vertical representation Demand, Supply & Allocations

Supply curve for good 1 and "demand"\* for goods 1 and higher

\*"Demand" is auction price, i.e., (inverse) demand, adjusted for allocation of bids to remaining goods, and, e.g., substituted by "reserve price" if actual bidding yields no sale

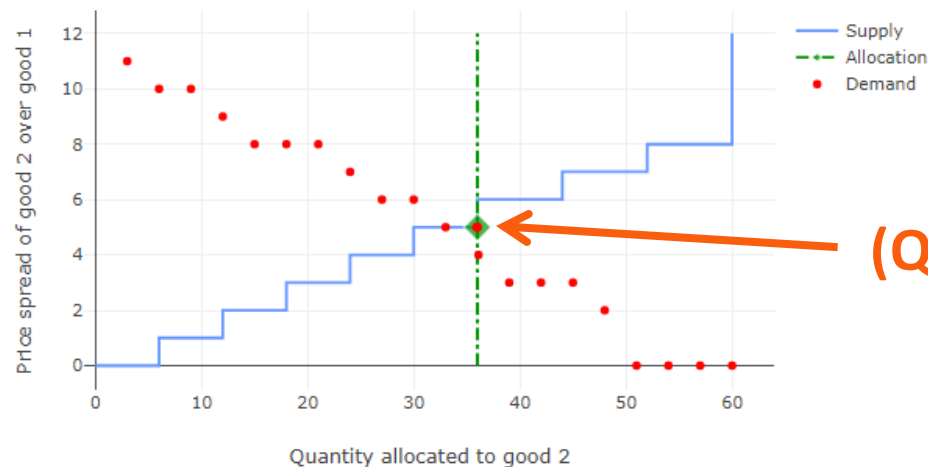


$$(Q_1 + Q_2, P_1) = (60, 9)$$



Supply curve for good 2 and "demand"\* for goods 2 and higher

\*"Demand" is auction price, i.e., (inverse) demand, adjusted for allocation of bids to remaining goods, and, e.g., substituted by "reserve price" if actual bidding yields no sale



$$(Q_1, P_1) = (24, 9)$$

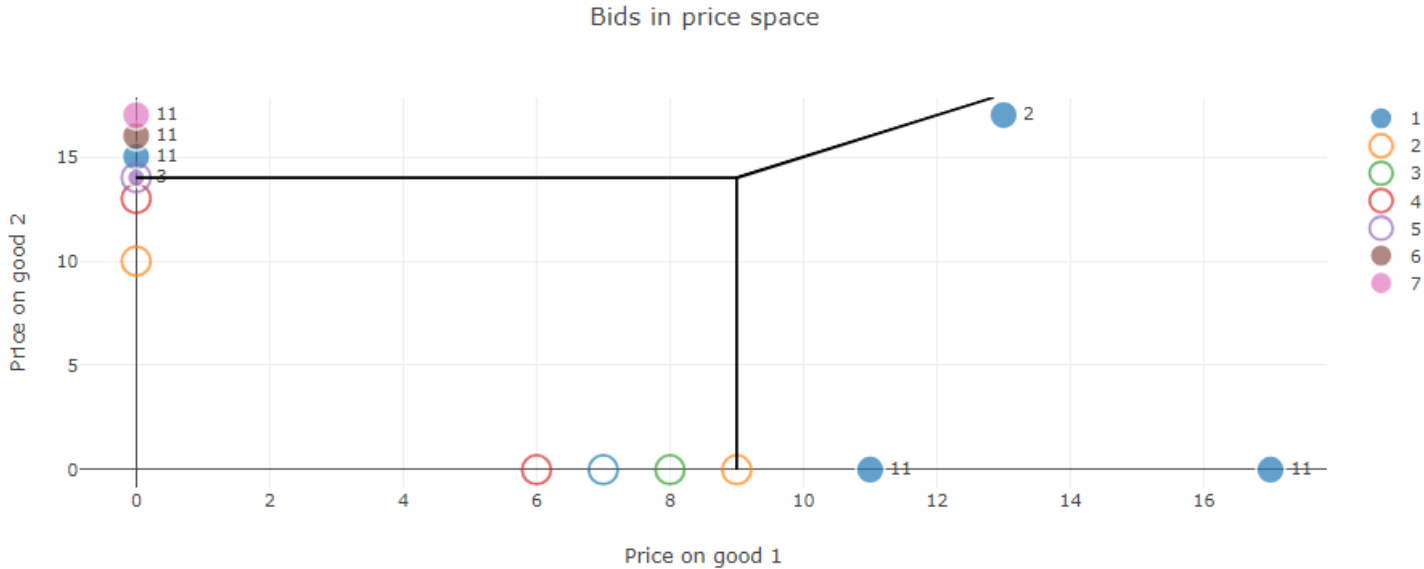
$$(Q_2, P_2) = (36, 14)$$



$$(Q_2, P_2 - P_1) = (36, 5)$$

# 3.1 Vertical representation

## Bids in price space & Allocations of bids to goods

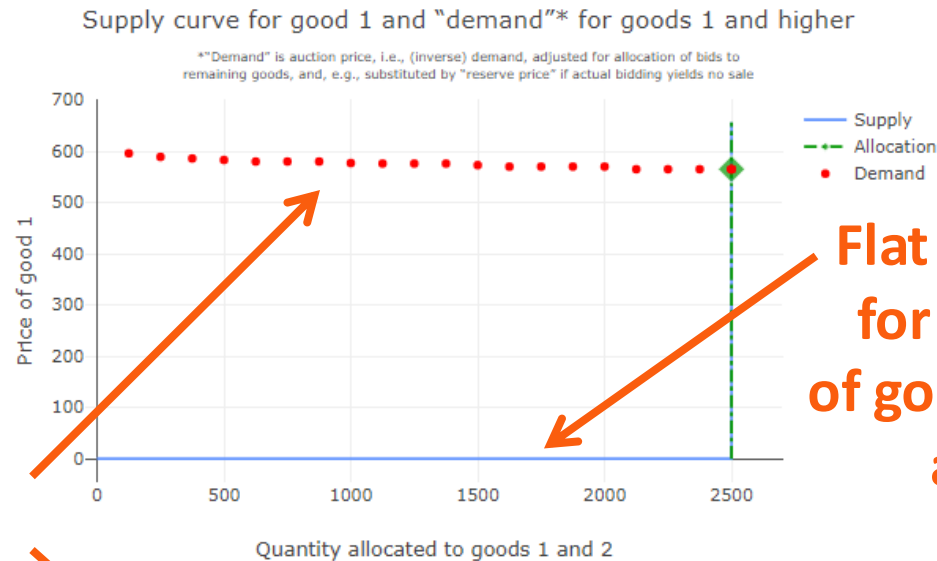


## Section 3.2

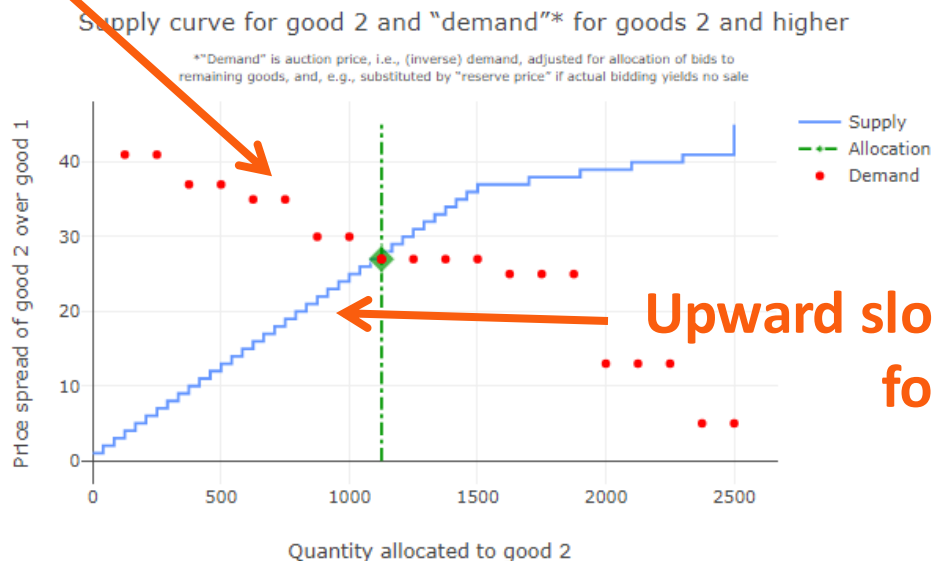
# 3. The “Vertical” Representation, and the Bank of England’s Auction

## 3.2 The Bank of England’s Indexed Long-Term Repo Auction

Demand curves  
are created by  
various paired  
and single bids  
for good 1  
OR good 2



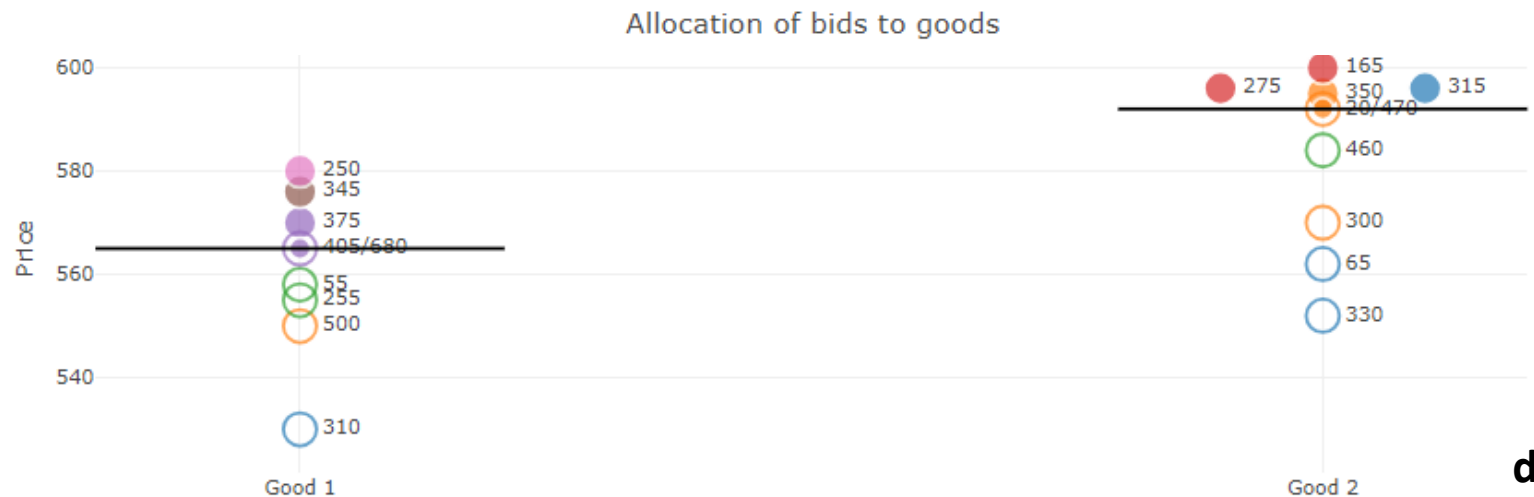
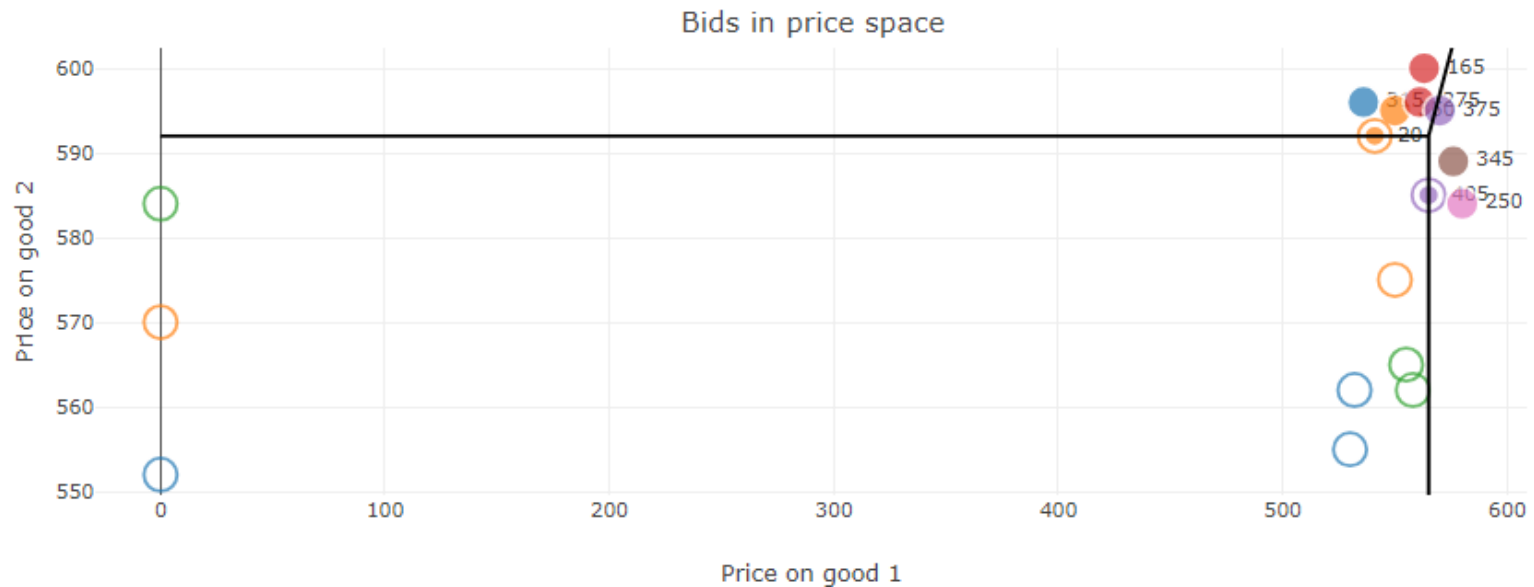
Flat supply of 2500  
for total quantity  
of good 1 and good 2  
at zero cost



Upward sloping supply curve  
for good 2

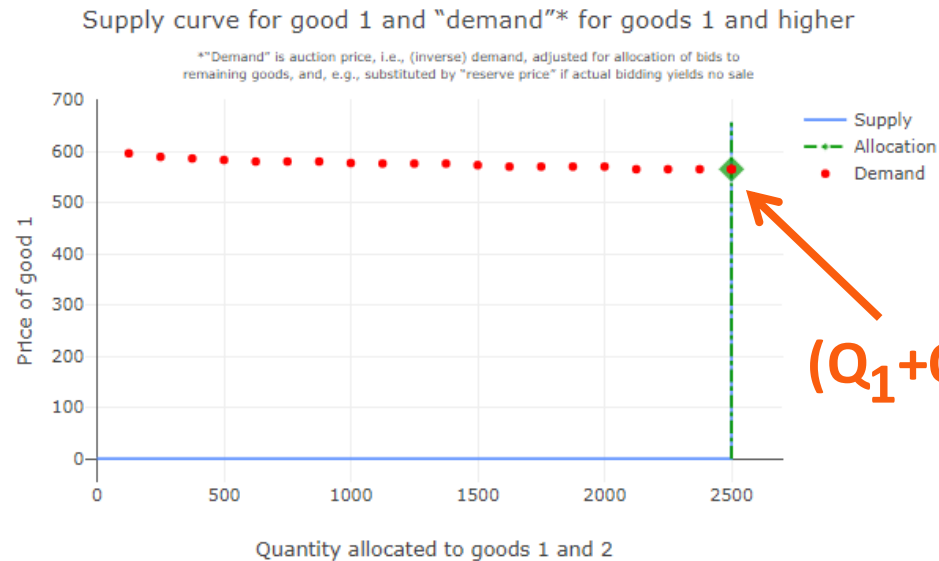
## 3.2 The Bank of England's ILTR Auction (Initial Implementation)

### Bids in price space & Allocations of bids to goods



## 3.2 The Bank of England's ILTR Auction (Initial Implementation)

### Demand, Supply, & Allocations



$$(Q_1 + Q_2, P_1) = (2500, 565)$$



$$(Q_1, P_1) = (1375, 565)$$

$$(Q_2, P_2) = (1125, 592)$$



$$(Q_2, P_2 - P_1) = (1125, 27)$$

# 3.2 The Bank of England's ILTR Auction (Initial Implementation)

## More Stressed Demand

Supply curve for good 1 and "demand"\* for goods 1 and higher

\*\*Demand\* is auction price, i.e., (inverse) demand, adjusted for allocation of bids to remaining goods, and, e.g., substituted by "reserve price" if actual bidding yields no sale

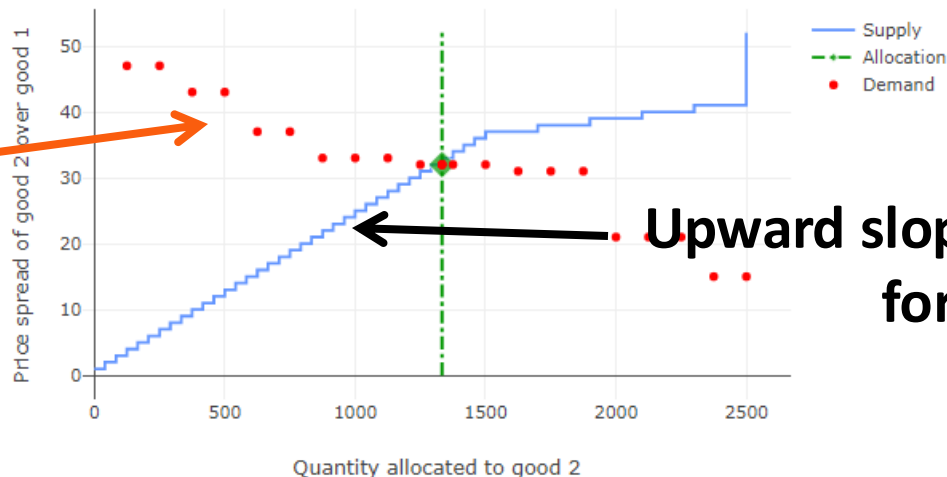


Various paired and single bids for good 1 OR good 2

Flat supply of 2500 for total quantity of good 1 and good 2 at zero cost

Supply curve for good 2 and "demand"\* for goods 2 and higher

\*\*Demand\* is auction price, i.e., (inverse) demand, adjusted for allocation of bids to remaining goods, and, e.g., substituted by "reserve price" if actual bidding yields no sale

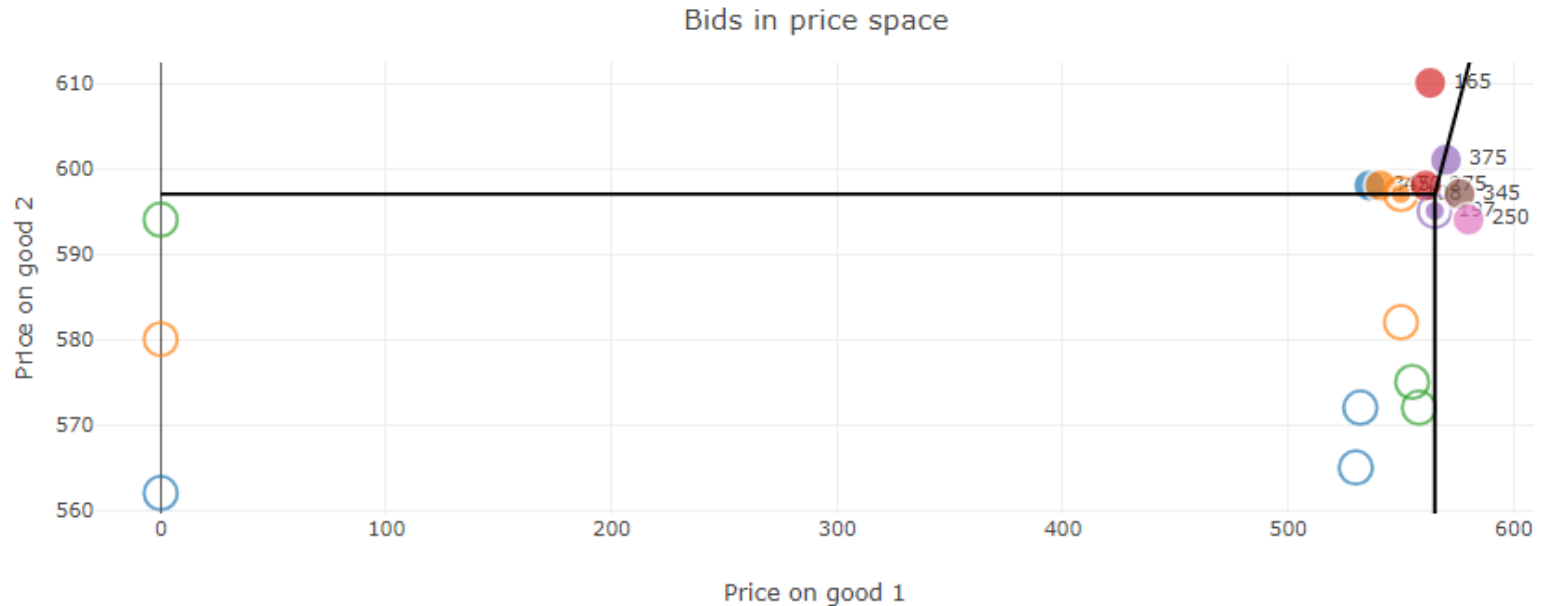


Increased prices for bids on good 2

Upward sloping supply curve for good 2

## 3.2 The Bank of England's ILTR Auction (Initial Implementation)

### More Stressed Demand

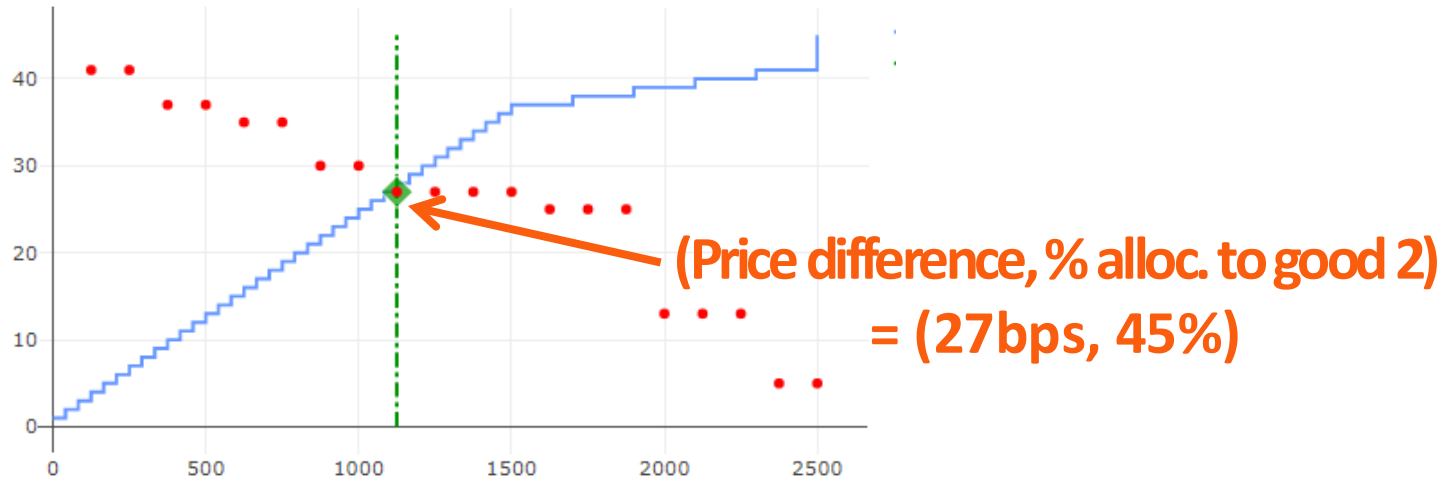




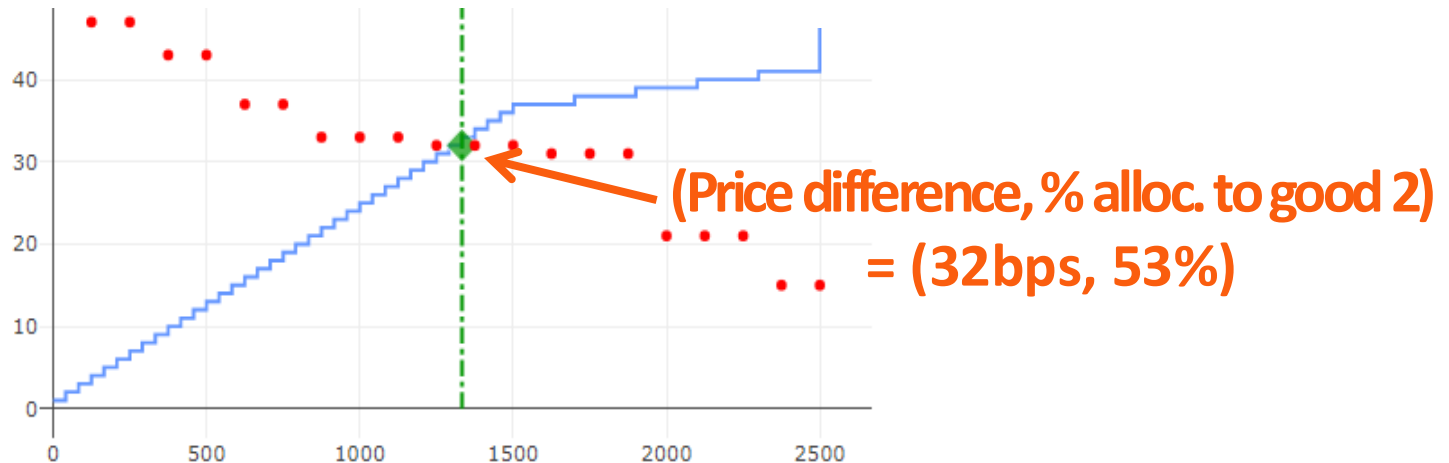
## 3.2 The Bank of England's ILTR Auction (Initial Implementation)

Total allocation capped at £2,500m

Less  
Stressed  
Demand



More  
Stressed  
Demand



## 3.2 The Bank of England's Indexed Long-Term Repo Auction

### BoE's Current Implementation with (Flexible) TQSS

TQSS Steps

Units	Price	
2500 2500	0 0	
1000 1000	120 120	×
800 800	190 190	×
700 700	250 250	×
400 400	330 330	×
500 500	380 380	×
600 600	460 460	×
400 400	520 520	×
500 500	580 580	×
200 200	630 630	×
400 400	690 690	×
300 300	750 750	×
500 500	900 900	×

Without TQSS, Quantity Cap = 2500

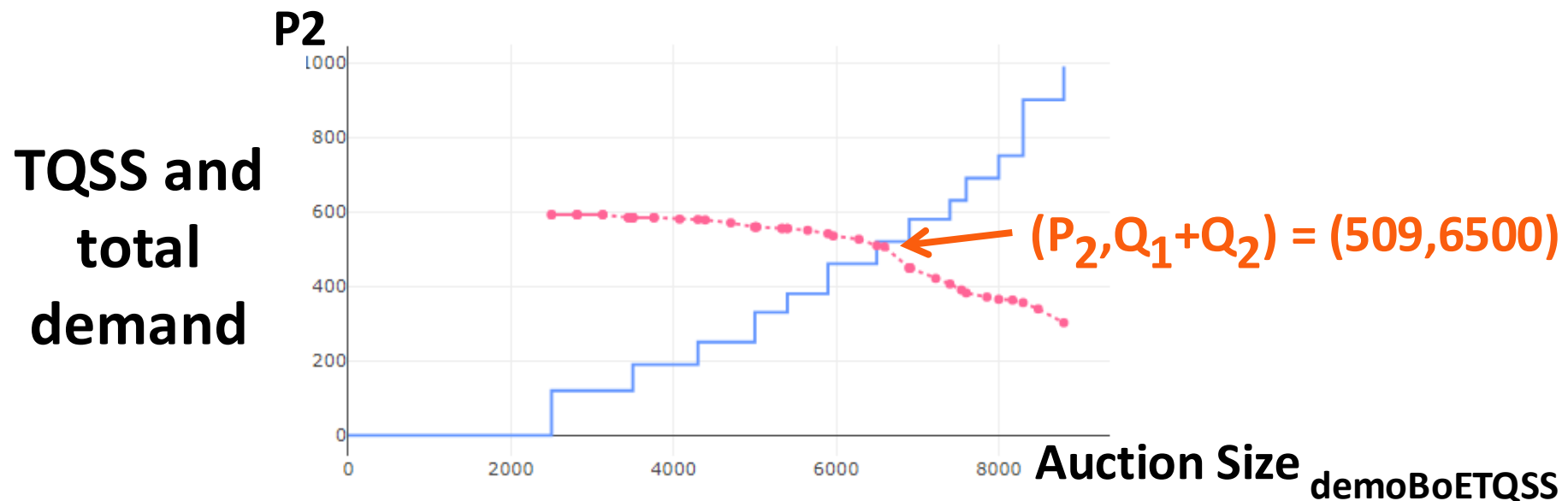
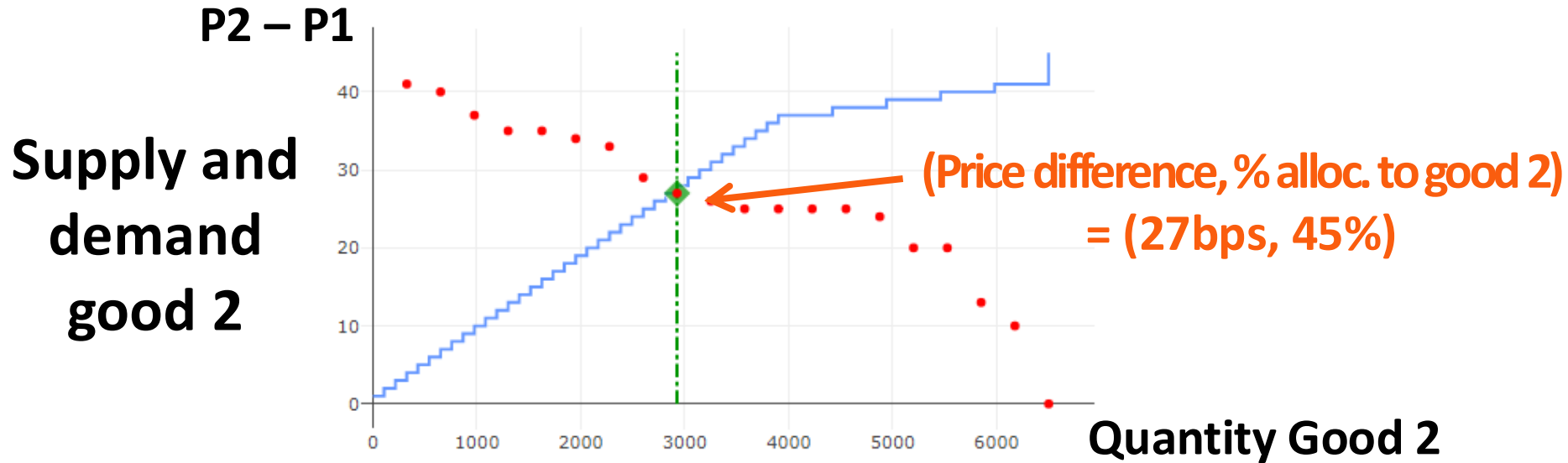
Implicit TQSS step with  
length = Supply curve of good 1 and higher

With additional steps above TQSS,  
Quantity Sold is  
function of price measure  
(here, function of price of good 2)

When demand is high,  
supply curves are scaled up until  
auction prices coincide with the  
TQSS,  
i.e. total demand and TQSS intersect

# 3.2 The Bank of England's ILTR Auction (Current Implementation)

## Demand, Supply & Allocations & TQSS



## Section 4

## 4. More Examples for Horizontal Applications, e.g., Related Bonds – Limited Flexibility Between Varieties

Supply curves for each good

Supply curve for good 1

Units	Price
<input type="text" value="35"/>	<input type="text" value="0"/>

35 [was 60]

Supply curve for good 2

Units	Price
<input type="text" value="35"/>	<input type="text" value="0"/>

35 [was 60]

TQSS

TQSS enabled

Enabled

Price normalisation and scaling

Normalised - constrain supply (may constrain quantities below supply cu

TQSS Steps

Units	Price
<input type="text" value="60"/>	<input type="text" value="0"/>

# 4. Limited Flexibility Between Varieties

## Demand, Supply & Allocations

Supply curves for each good

Supply curve for good 1   Supply curve

Units	Price
<input type="text" value="35"/>	<input type="text" value="0"/>

Supply curve for good 2   Supply curve

Units	Price
<input type="text" value="35"/>	<input type="text" value="0"/>

TQSS

TQSS enabled

Enabled

Price normalisation and scaling

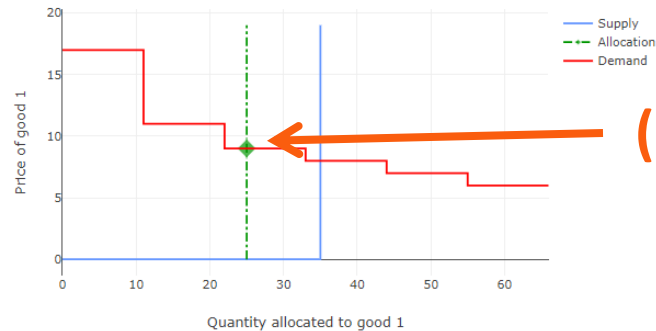
Normalised - constrain supply (may constrain quantities below supply curve)

TQSS Steps

Units	Price
<input type="text" value="60"/>	<input type="text" value="0"/>

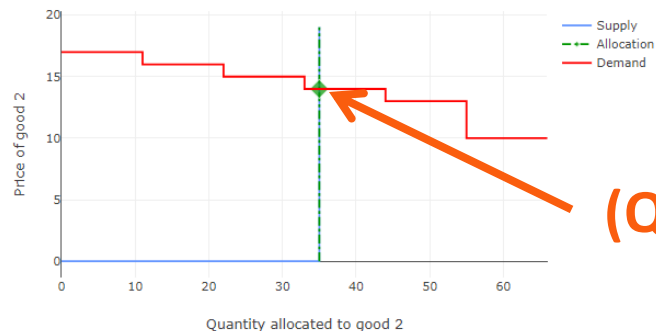
Supply curve and unadjusted demand\* for good 1

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



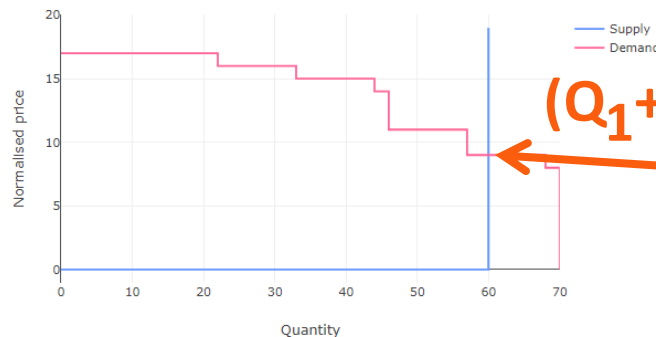
Supply curve and unadjusted demand\* for good 2

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



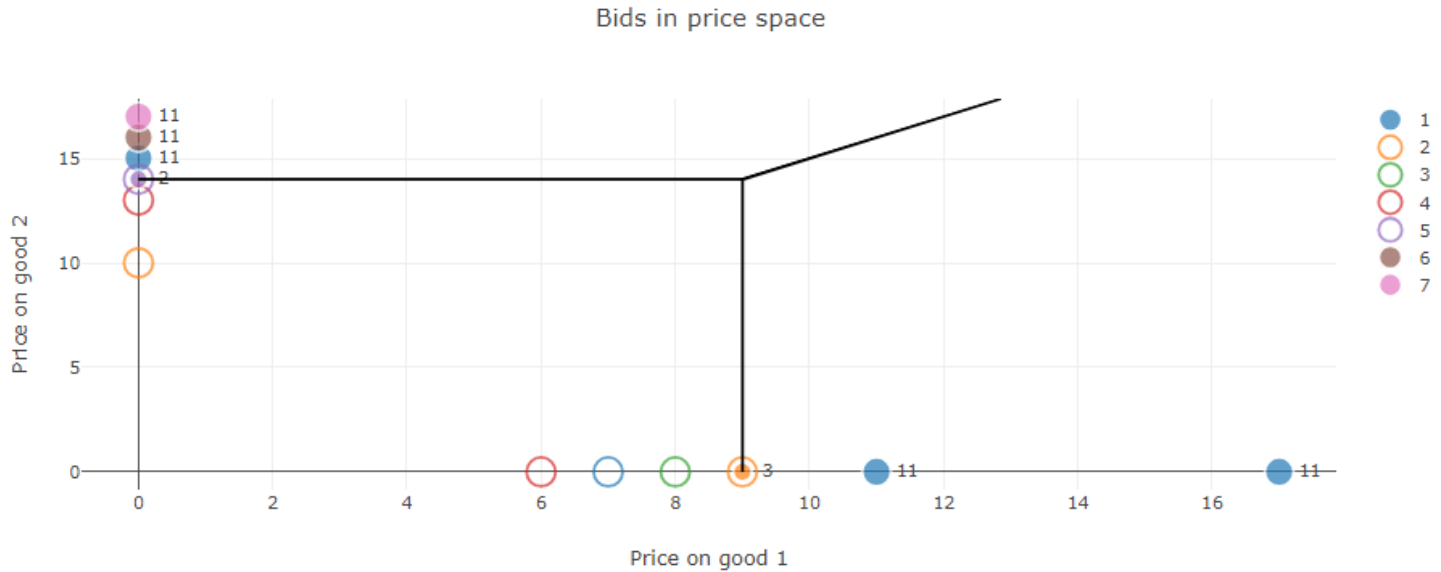
TQSS and (normalised) total demand

When some bids are paired, graph of demand is approximate for quantities exceeding total actually allocated.



# 4. Limited Flexibility Between Varieties

## Bids in price space & Allocations of bids to goods



# 4. More Examples for Horizontal Applications, e.g., Related Bonds – Additional Flexibility on Total Sales

Supply curve for good 1

Units

Price

35

0

Supply curve for good 2

Units

Price

35

0

TQSS

TQSS enabled

Enabled

Price normalisation and scaling

Normalised - constrain supply (may constrain quantities below supply curves)

TQSS Steps

Units	Price	
50	0	✕
5	5	✕
5	10	✕

50 [was 60]



# 4. More Examples for Horizontal Applications, e.g., Closely Related Bonds – Additional Flexibility on Total Sales

Supply curve for good 1

Units	Price
35	0

Supply curve for good 2

Units	Price
35	0

TQSS

TQSS enabled

Enabled

Price normalisation and scaling

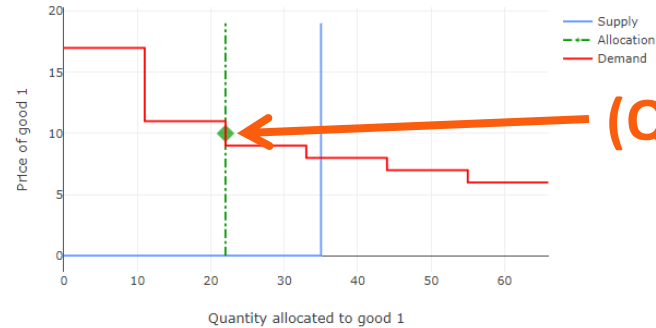
Normalised - constrain supply (may constrain quantities below supply curves)

TQSS Steps

Units	Price	
50	0	50 [was 60]
5	5	
5	10	

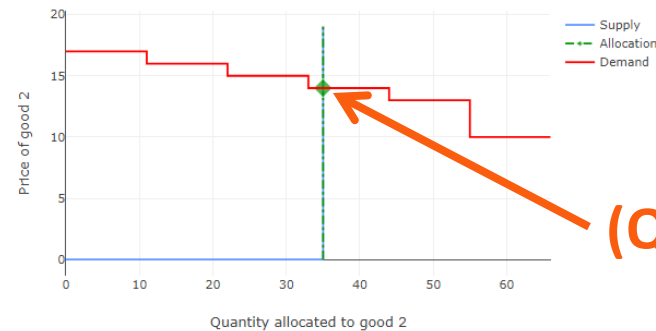
Supply curve and unadjusted demand\* for good 1

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



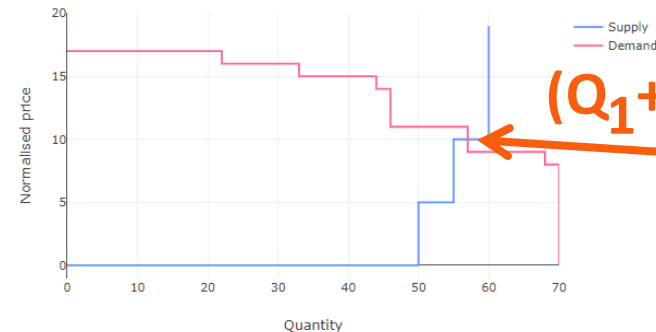
Supply curve and unadjusted demand\* for good 2

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



TQSS and (normalised) total demand

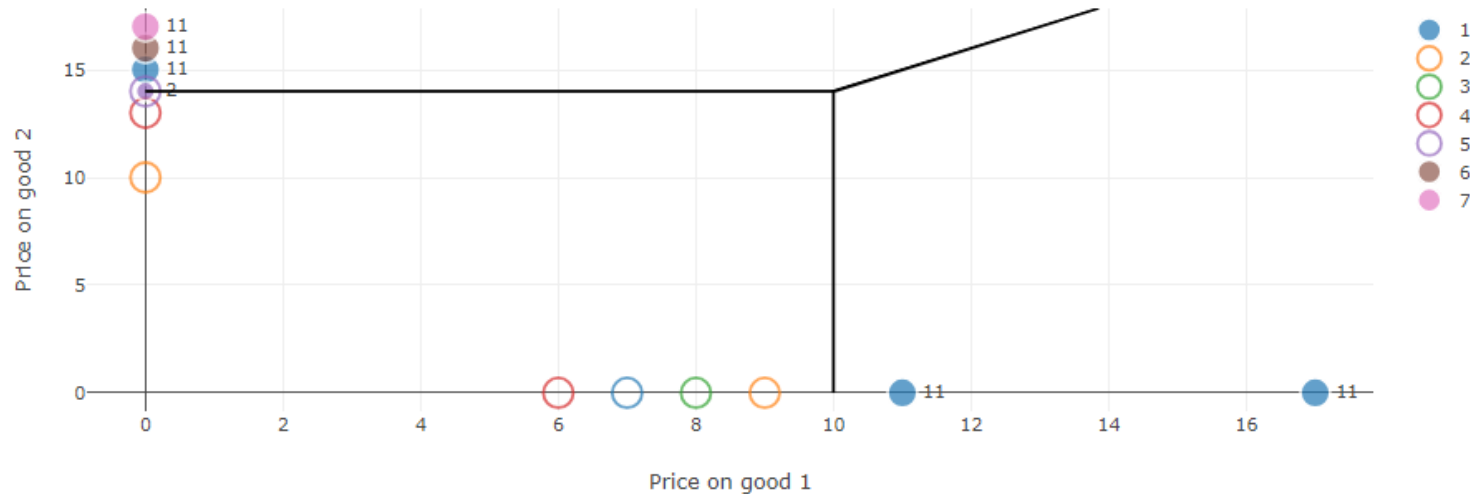
When some bids are paired, graph of demand is approximate for quantities exceeding total actually allocated.



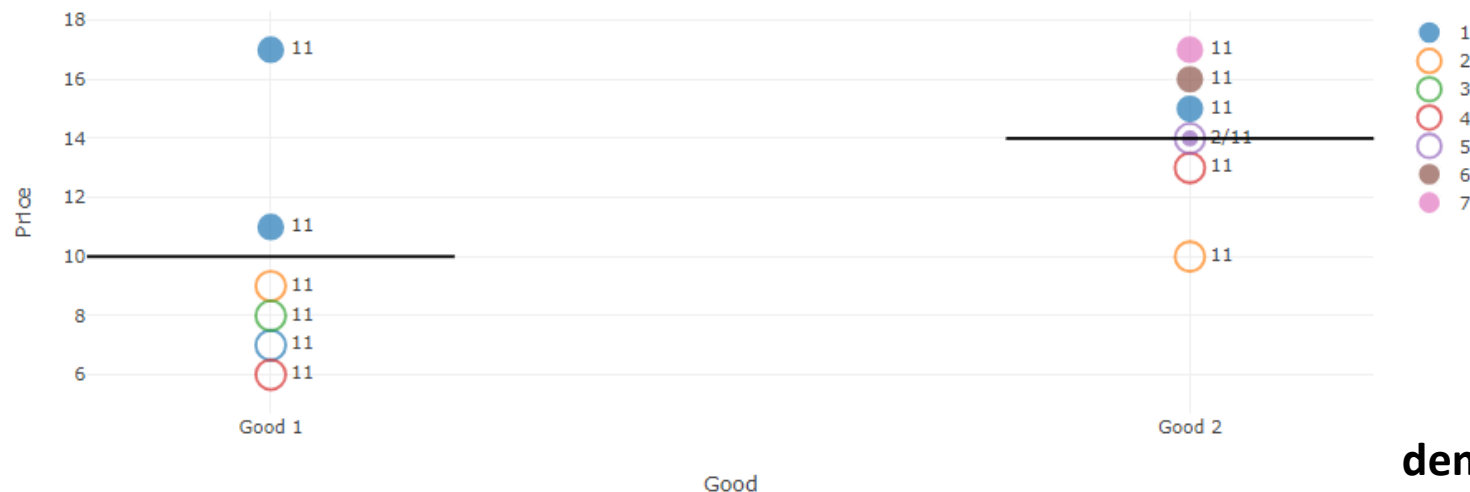
# 4. Additional Flexibility on Total Sales

## Bids in price space & Allocations of bids to goods

Bids in price space



Allocation of bids to goods



# 4. More Examples for Horizontal Applications, e.g., Related Bonds – More Flexible Auctioneer

Supply curves for each good

Supply curve for good 1

Units	Price	
20 <b>20 [was 35]</b> 0 <b>0</b>		×
15 <b>15</b>	5 <b>5</b>	×

Supply curve for good 2

Units	Price	
20 <b>20 [was 35]</b> 5 <b>5 [was 0]</b>		×
15 <b>15</b>	8 <b>8</b>	×

TQSS

TQSS Steps

Units	Price	
50	0	×
5	5	×
5	10	×

# 4. More Examples for Horizontal Applications, e.g., Related Bonds – More Flexible Auctioneer

Supply curves for each good

Supply curve for good 1  Supply curve

Units	Price	
20 <b>20 [was 35]</b> 0 0		<input type="button" value="x"/>
15 <b>15</b>	5 <b>5</b>	<input type="button" value="x"/>

Supply curve for good 2  Supply curve

Units	Price	
20 <b>20 [was 35]</b> 5 5 <b>[was 0]</b>		<input type="button" value="x"/>
15 <b>15</b>	8 <b>8</b>	<input type="button" value="x"/>

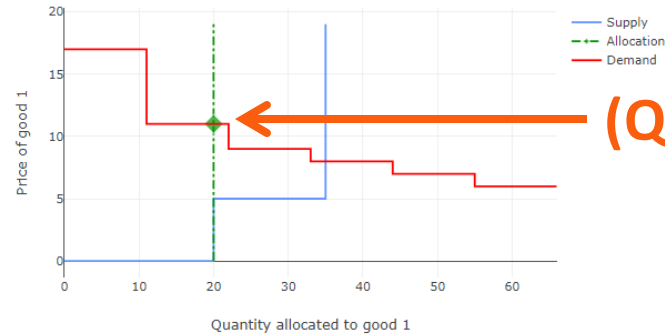
TQSS

TQSS Steps

Units	Price	
50	0	<input type="button" value="x"/>
5	5	<input type="button" value="x"/>
5	10	<input type="button" value="x"/>

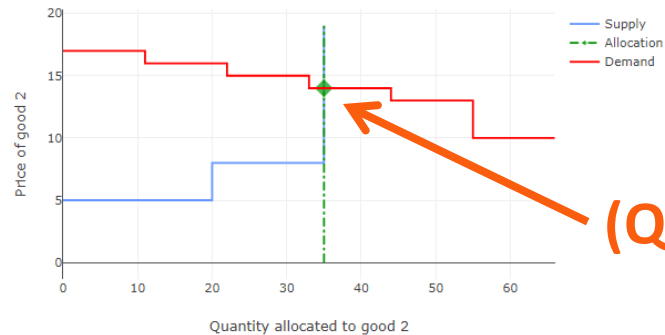
Supply curve and unadjusted demand\* for good 1

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



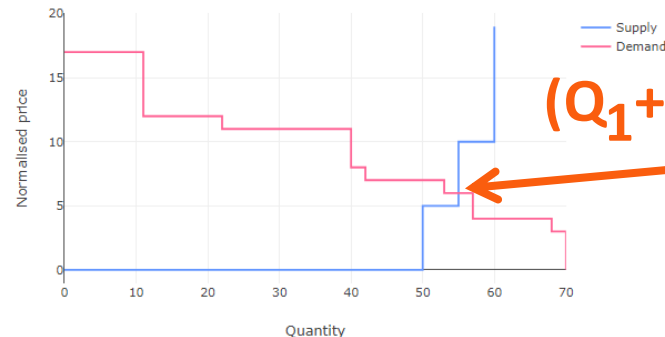
Supply curve and unadjusted demand\* for good 2

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



TQSS and (normalised) total demand

When some bids are paired, graph of demand is approximate for quantities exceeding total actually allocated.



demoBonds3

# 4. More Flexible Auctioneer

## Bids in price space & Allocations of bids to goods



# Section 5

## 5. More Examples of Paired Bids

Bidder 1 ▼ ✕ Bidder

Bid	Units	Price	Price	
1	11	17	0	✕
2	11	11	0	✕
3	11	7	0	✕
4	11	0	15	✕
5	2	15	17	✕

✕ Bid

Bidder 2 ▼ ✕ Bidder

Bid	Units	Price	Price	
1	11	9	0	✕
2	11	0	10	✕

✕ Bid

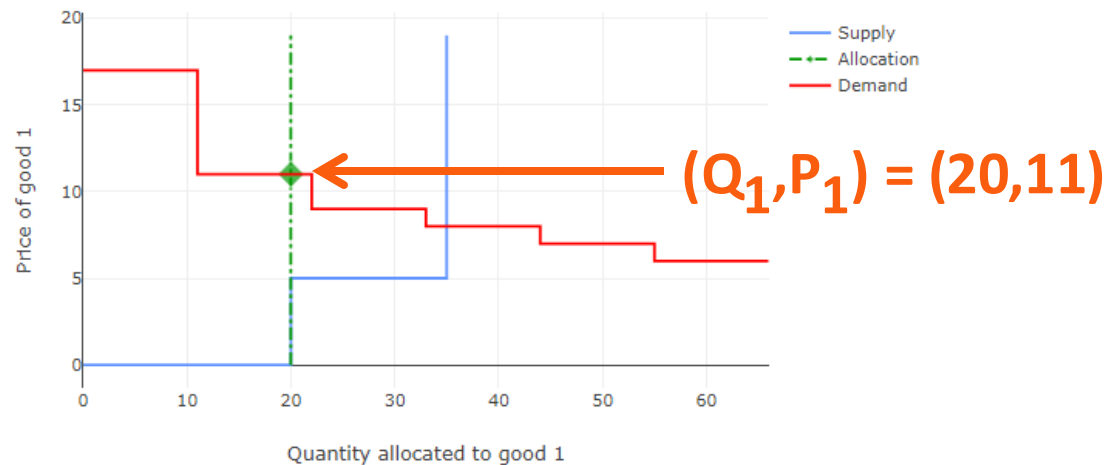
Bidder 3 ▼ ✕ Bidder

Bid	Units	Price	Price
1	11	8	0

# 5. More Examples of Paired Bids Demand, Supply & Allocations

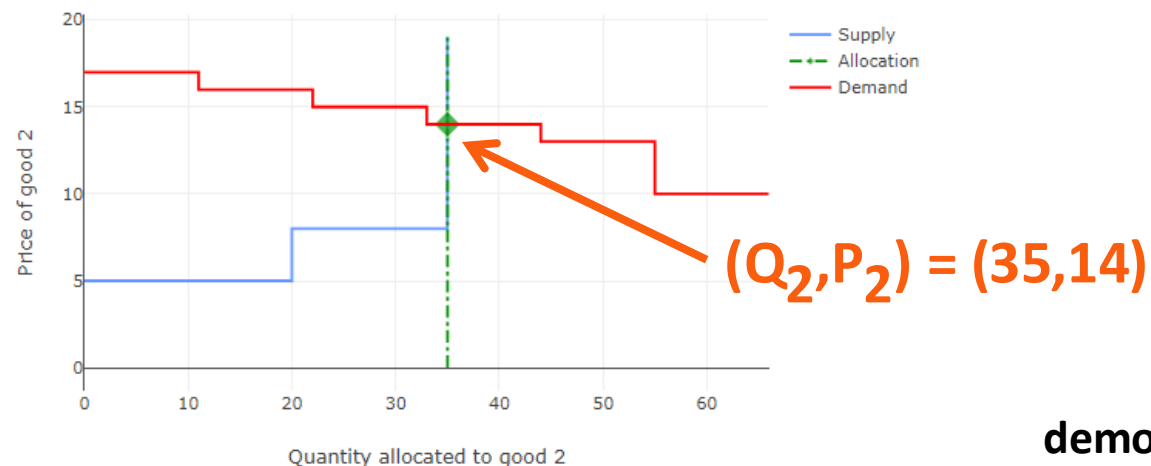
Supply curve and unadjusted demand\* for good 1

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



Supply curve and unadjusted demand\* for good 2

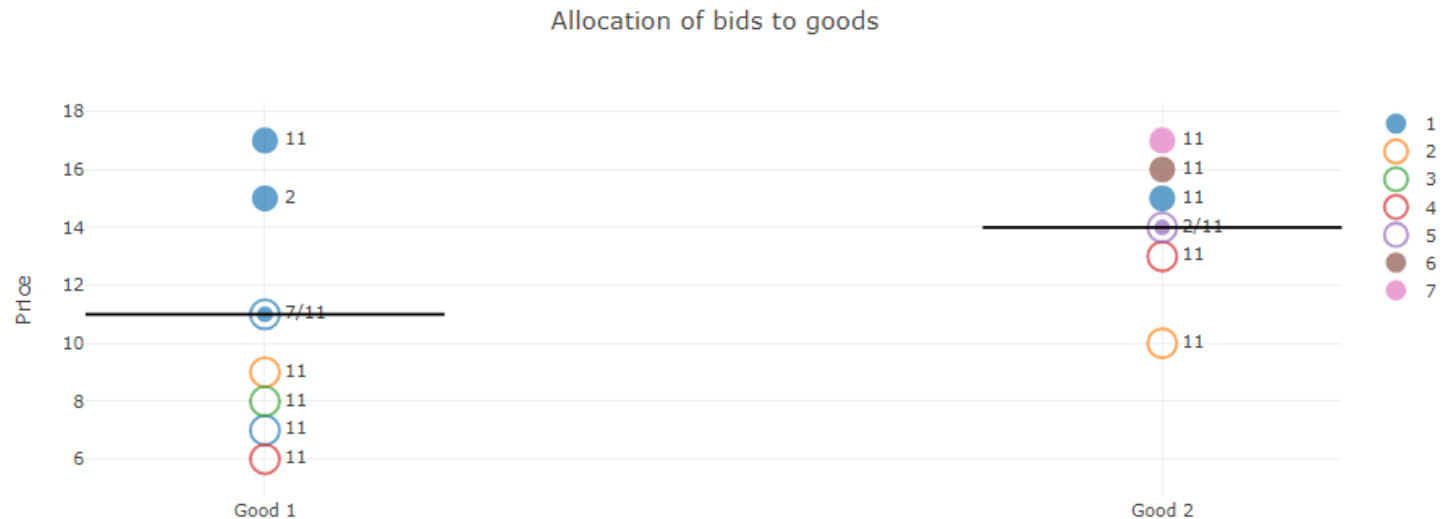
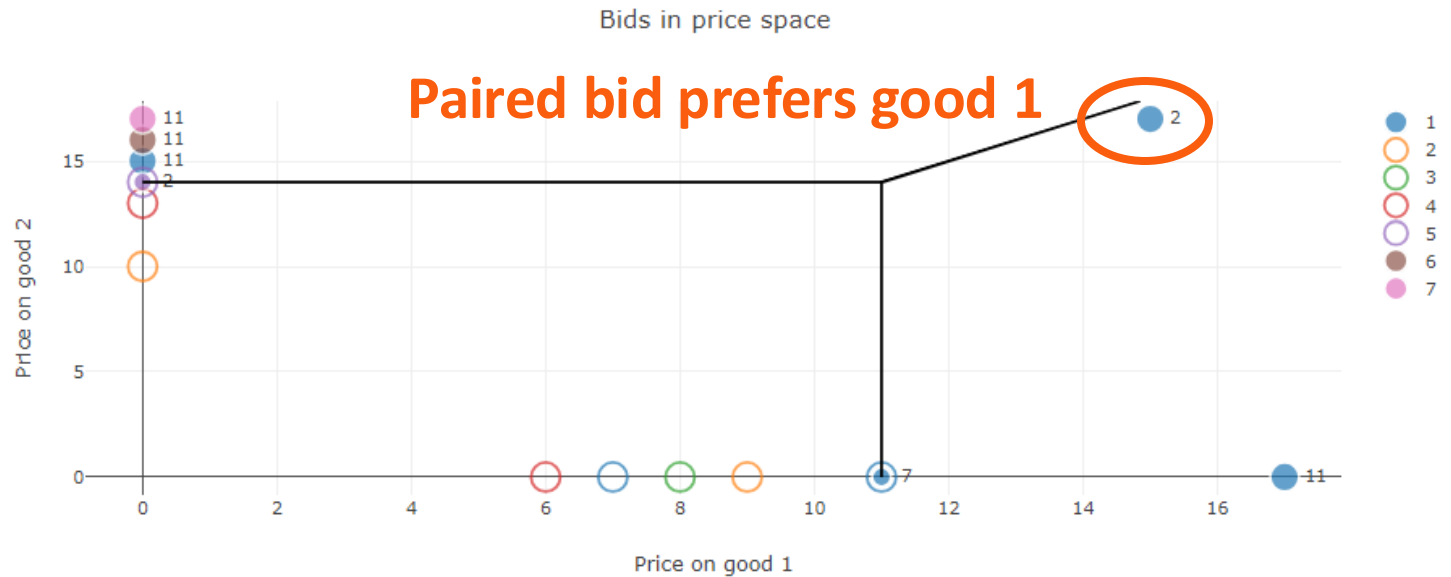
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods





# 5. More Examples of Paired Bids

## Bids in price space & Allocations of bids to goods



## 5. More Examples of Paired Bids

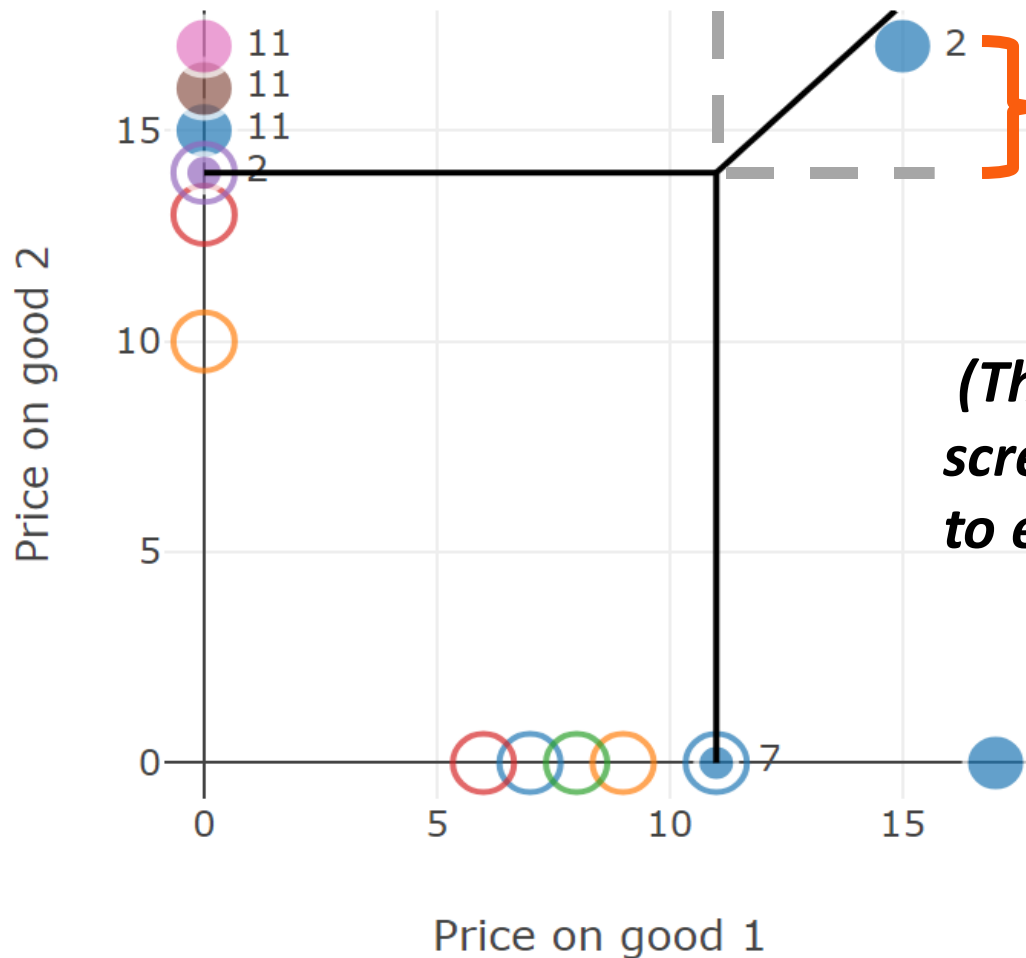
### Bids in price space

Bidder's surplus =  $15 - 11 = 4$

Bidder's surplus

=  $17 - 14$

= 3



## 5. More Examples of Paired Bids

Paired bid is indifferent between goods

Bidder 1

✕ Bidder

Bid	Units	Price	Price	
1	11	17	0	✕
2	11	11	0	✕
3	11	7	0	✕
4	11	0	15	✕
5	2	15	18	✕

✚ Bid

Bidder 2

✕ Bidder

Bid	Units	Price	Price	
1	11	9	0	✕
2	11	0	10	✕

✚ Bid

Bidder 3

✕ Bidder

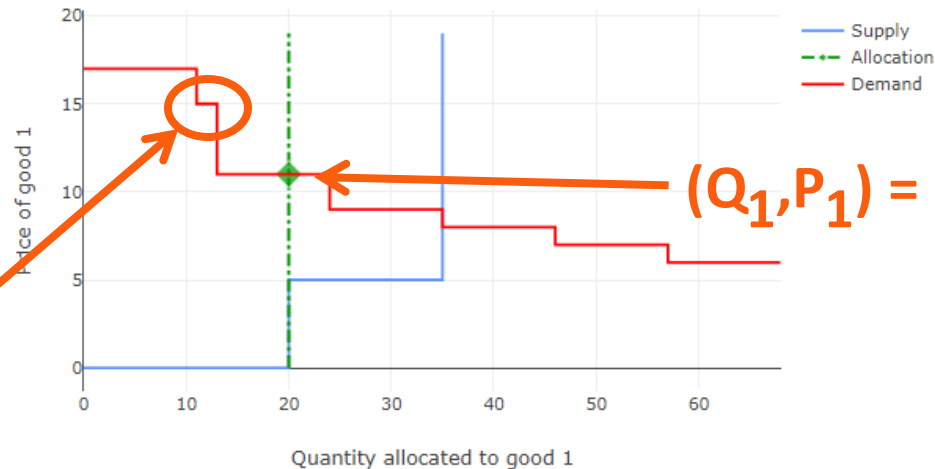
Bid	Units	Price	Price	
1	11	8	0	

# 5. Paired bid is indifferent between goods

## Demand, Supply & Allocations

Supply curve and unadjusted demand\* for good 1

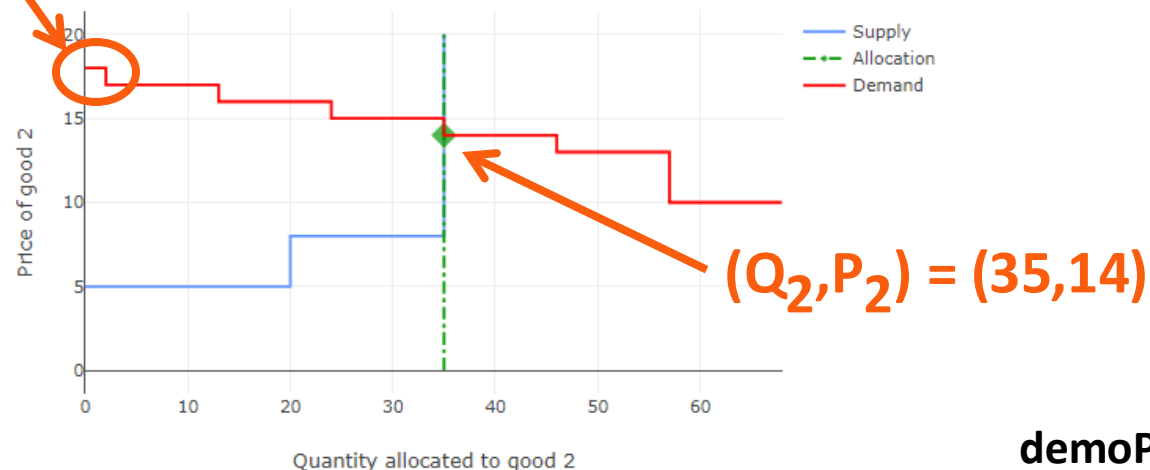
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



Paired bid  
[2,15,18]

Supply curve and unadjusted demand\* for good 2

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



# 5. Paired bid is indifferent between goods

## Bids in price space & Allocations of bids to goods

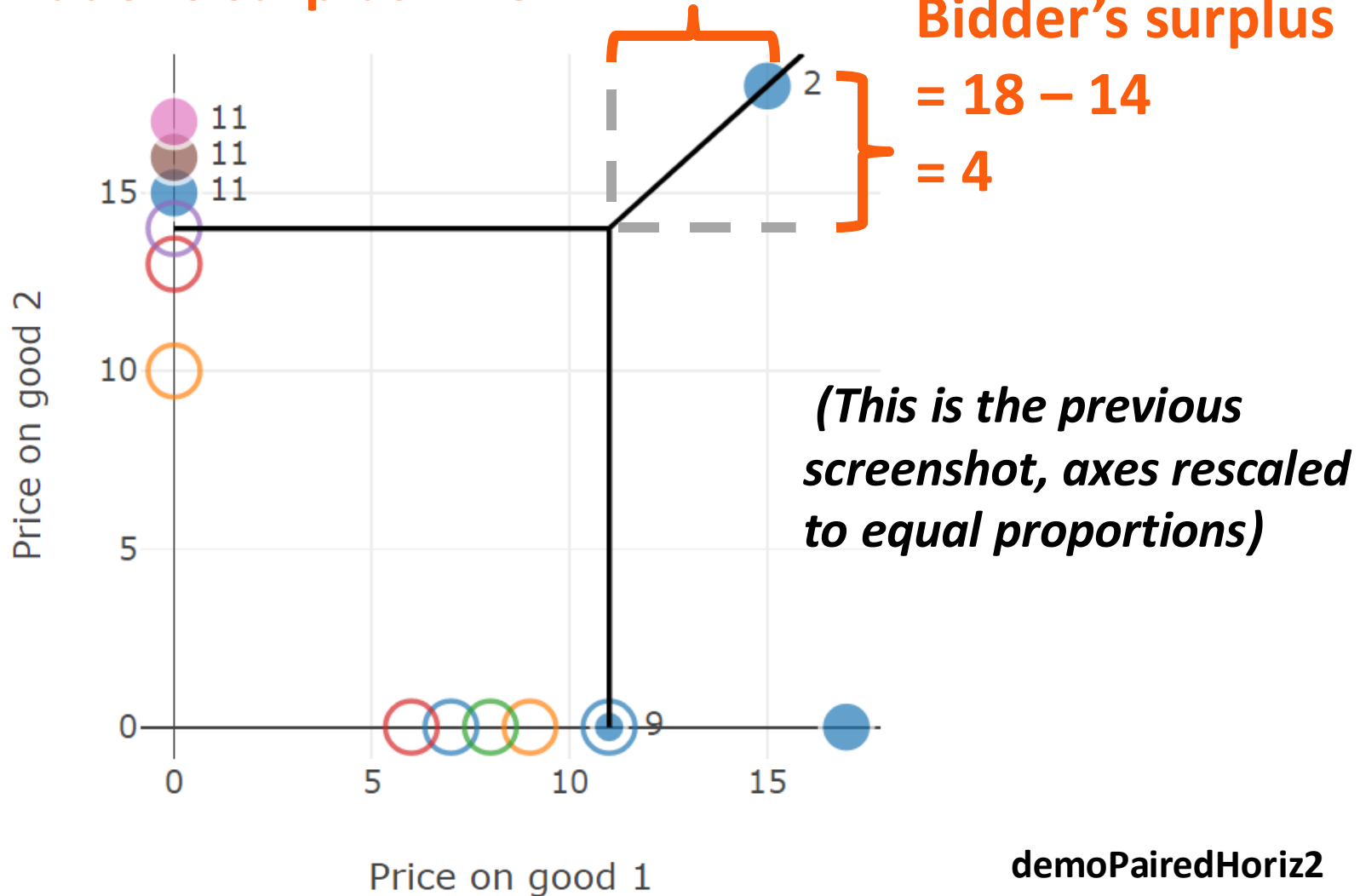


## 5. Paired bids is indifferent between goods

### Bids in price space

**Bidder's surplus =  $15 - 11 = 4$**

**Bidder's surplus**  
**= 18 – 14**  
**= 4**



## 5. More Examples of Paired Bids

### Paired bid is rationed between goods

Bidder 1

✕ Bidder

Bid	Units	Price	Price	
1	11	17	0	✕
2	11	11	0	✕
3	11	7	0	✕
4	11	0	15	✕
5	5	15	18	✕

✚ Bid

Bidder 2

✕ Bidder

Bid	Units	Price	Price	
1	11	9	0	✕
2	11	0	10	✕

✚ Bid

Bidder 3

✕ Bidder

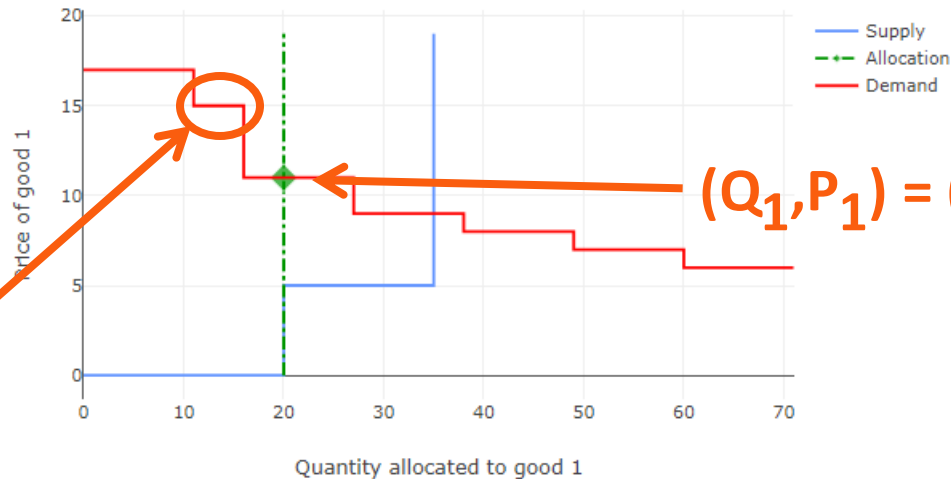
Bid	Units	Price	Price	
1	11	8	0	

# 5. Paired bid is rationed between goods

## Demand, Supply & Allocations

Supply curve and unadjusted demand\* for good 1

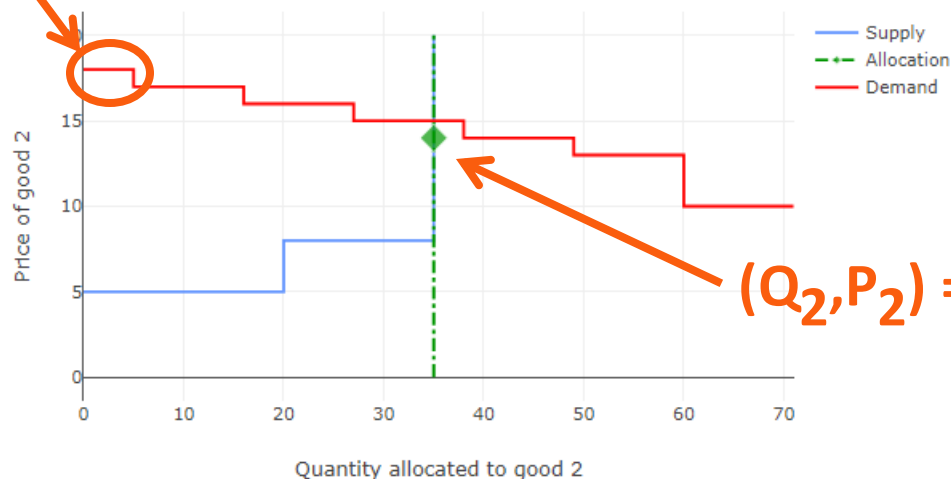
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$$(Q_1, P_1) = (20, 11)$$

Supply curve and unadjusted demand\* for good 2

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



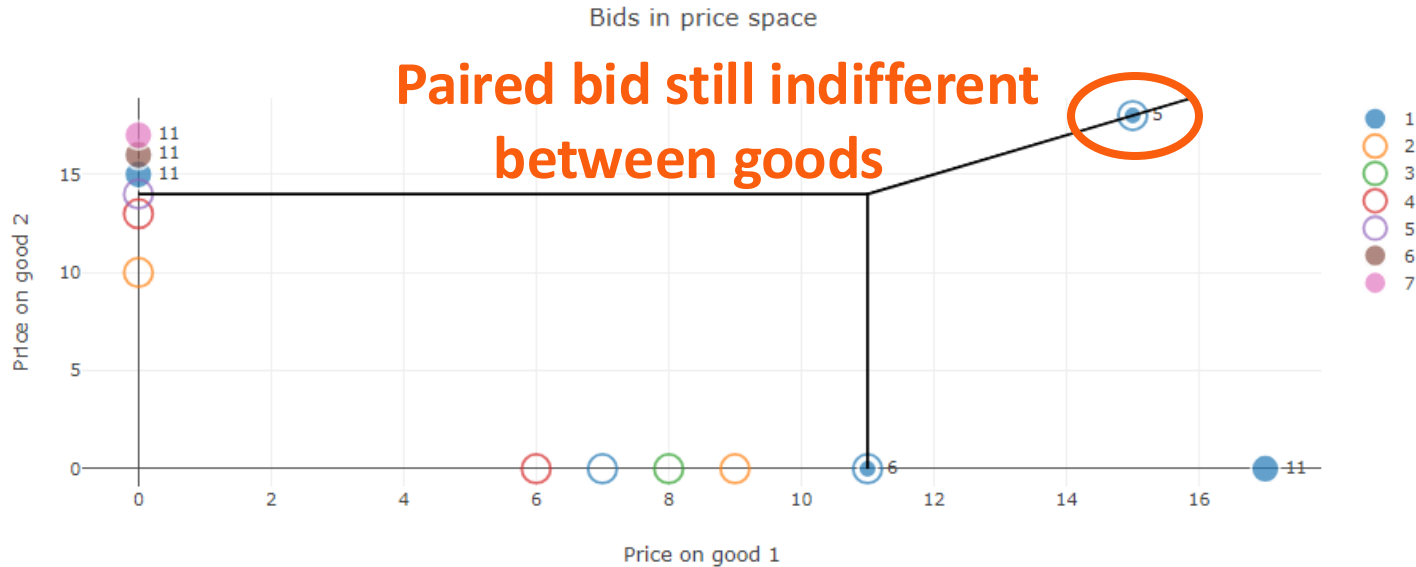
$$(Q_2, P_2) = (35, 14)$$

Paired bid  
[5,15,18]



# 5. Paired bid is rationed between goods

## Bids in price space & Allocations of bids to goods



## 5. Paired bid is rationed between goods

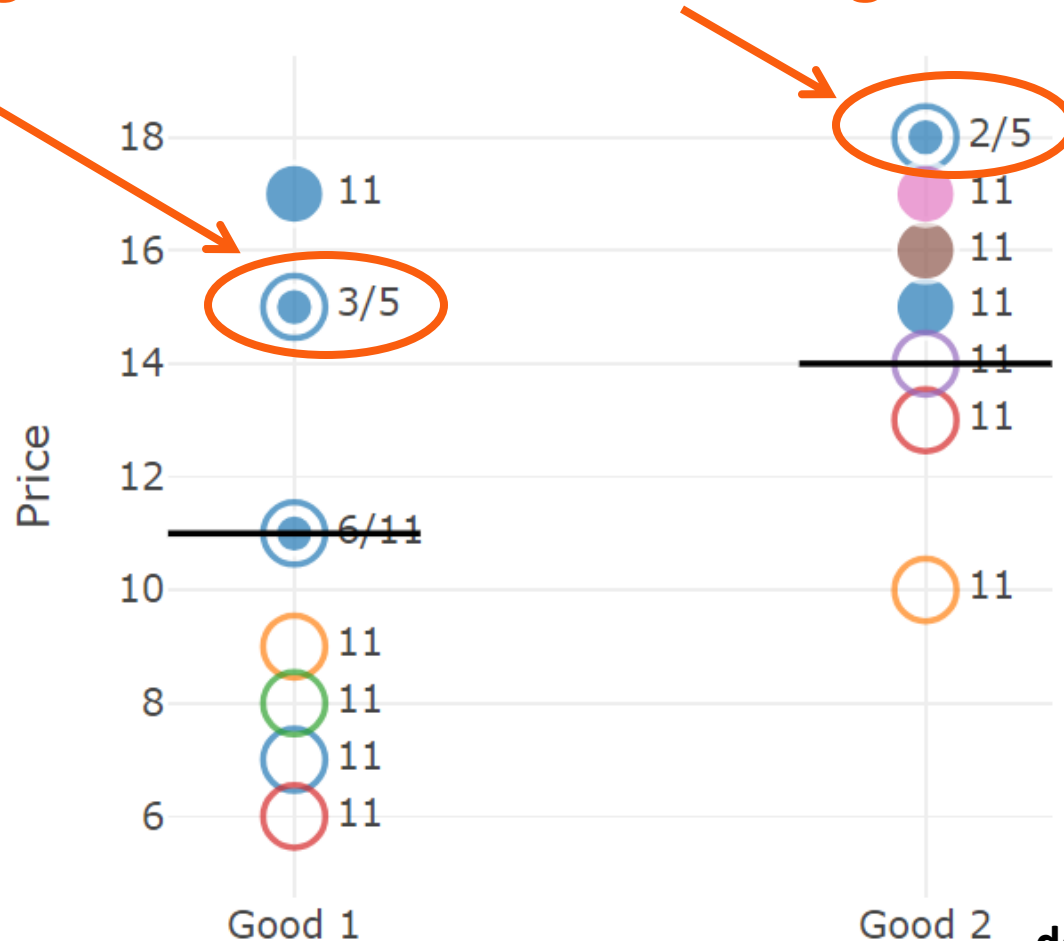
### Allocations of bids to goods

Paired bid (for 5 units in total) receives

3 units of good 1

AND

2 units of good 2



# 5. More Examples of Paired Bids

## Paired bid is rationed between goods and no sale

Bidder 1

✕ Bidder

Bid	Units	Price	Price	
1	11	17	0	✕
2	11	11	0	✕
3	11	7	0	✕
4	11	0	15	✕
5	50	12	15	✕

+ Bid

Bidder 2

✕ Bidder

Bid	Units	Price	Price	
1	11	9	0	✕
2	11	0	10	✕

+ Bid

Bidder 3

✕ Bidder

Bid	Units	Price	Price	
1	11	8	0	

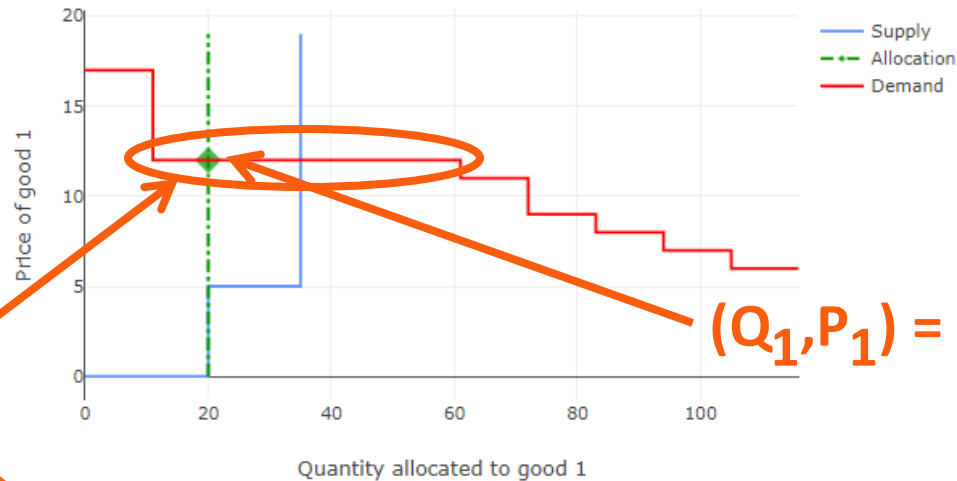
5 50 15 12 18 15

# 5. Paired bid is rationed between goods and no sale

## Demand, Supply & Allocations

Supply curve and unadjusted demand\* for good 1

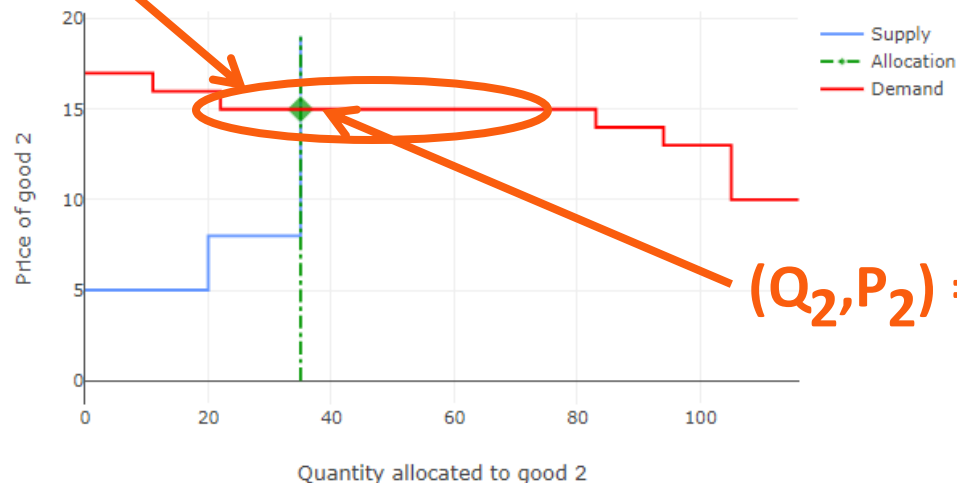
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$$(Q_1, P_1) = (20, 12)$$

Supply curve and unadjusted demand\* for good 2

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods

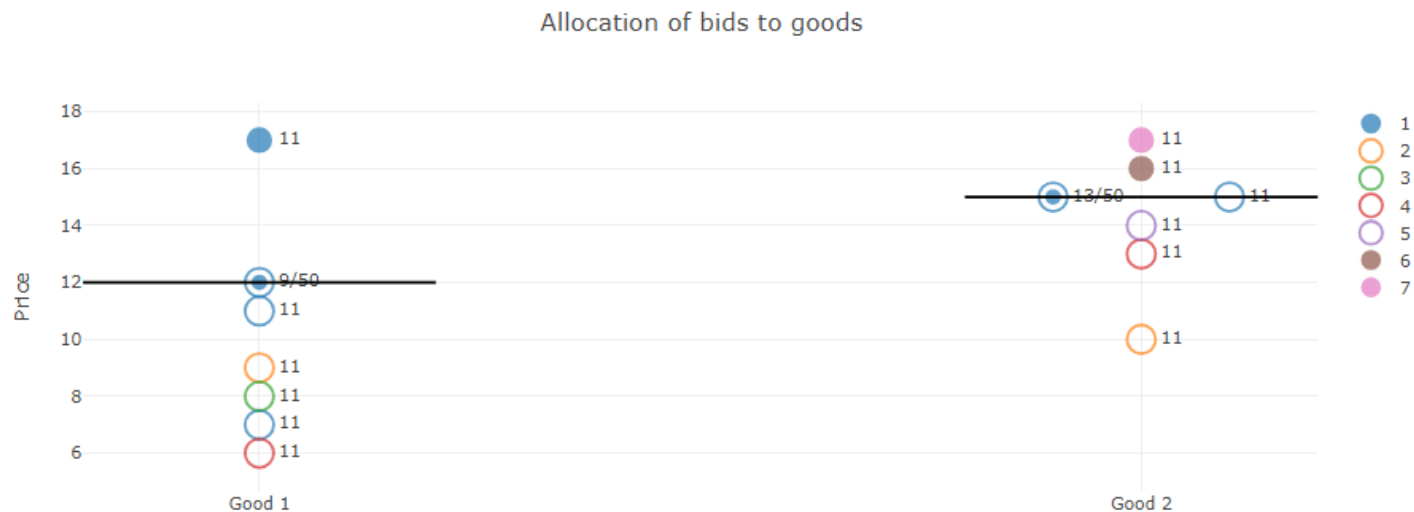
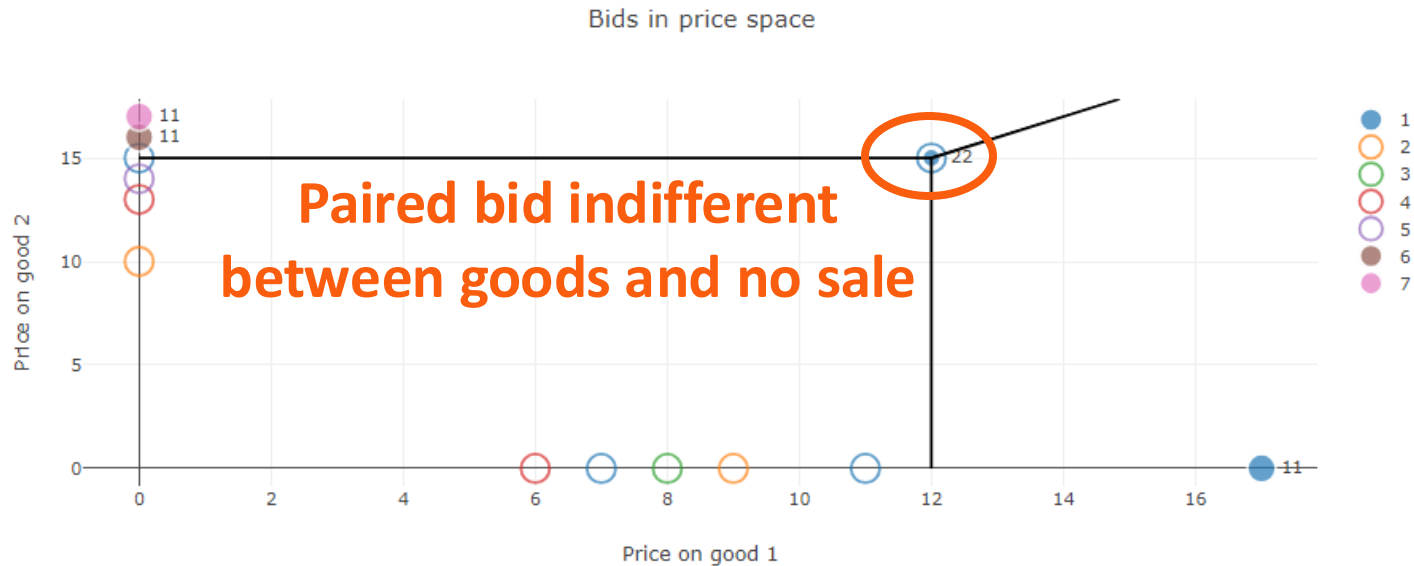


$$(Q_2, P_2) = (35, 15)$$

Paired bid  
[50,12,15]

# 5. Paired bid is rationed between goods and no sale

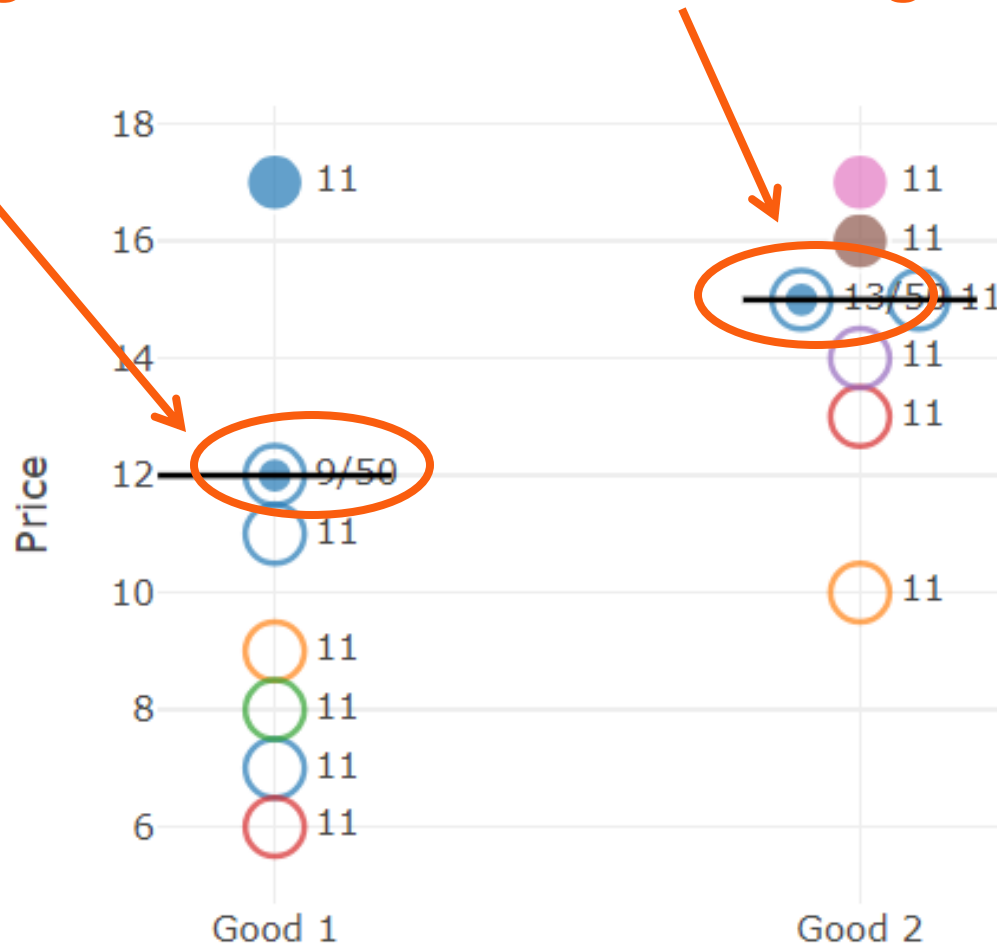
## Bids in price space & Allocations of bids to goods



## 5. Paired bid is rationed between goods and no sale

### Allocations of bids to goods

Paired bid (for 50 units in total) receives  
9 units of good 1 AND 13 units of good 2



## Section 6

## 6. More Than Two Varieties

Quantity  
bid for

Good 1  
Price bid

Good 2  
Price bid

Good 3  
Price bid

Good 4  
Price bid

Bidder 8   Bidder

Bid	Units	Price	Price	Price	Price	
1	11	14	13	11	0	×
2	11	12	14	9	0	×
3	11	8	11	8	0	×
4	11	0	0	11	12	×
5	11	0	0	14	16	×
6	11	0	14	16	16	×
7	11	0	12	16	0	×
8	11	0	0	18	19	×

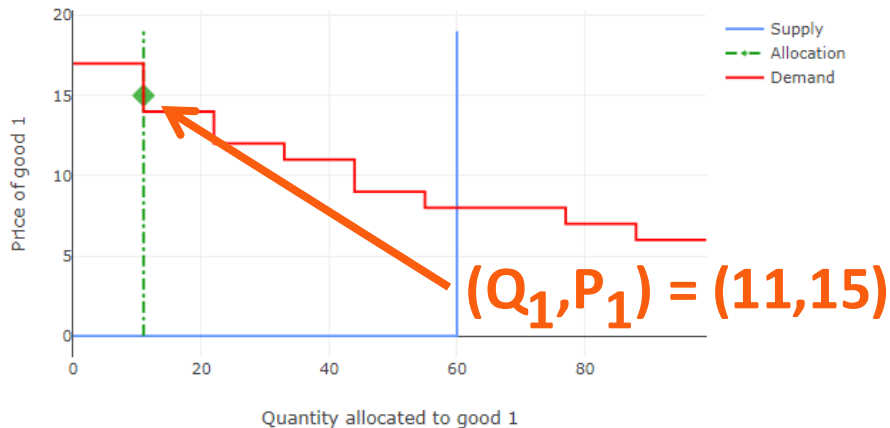


# 6. More Than Two Varieties

## Demand, Supply & Allocations

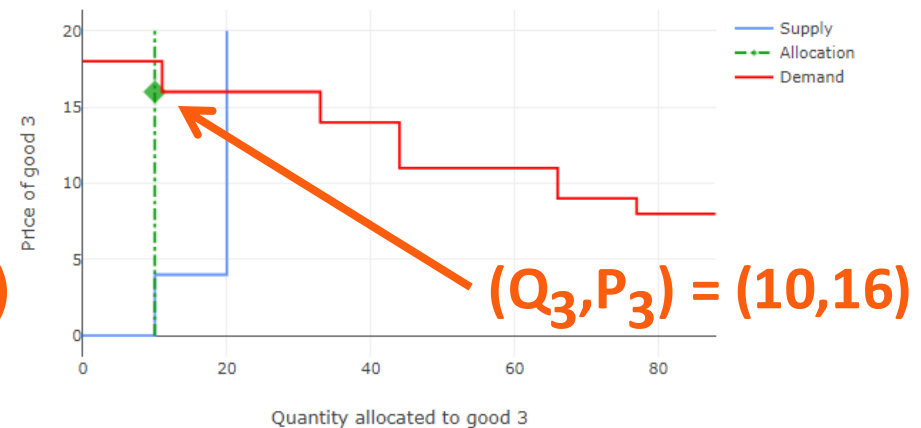
Supply curve and unadjusted demand\* for good 1

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



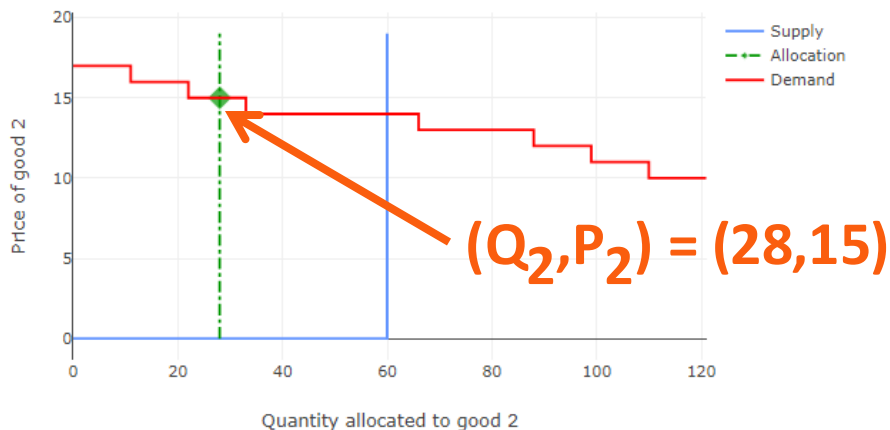
Supply curve and unadjusted demand\* for good 3

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



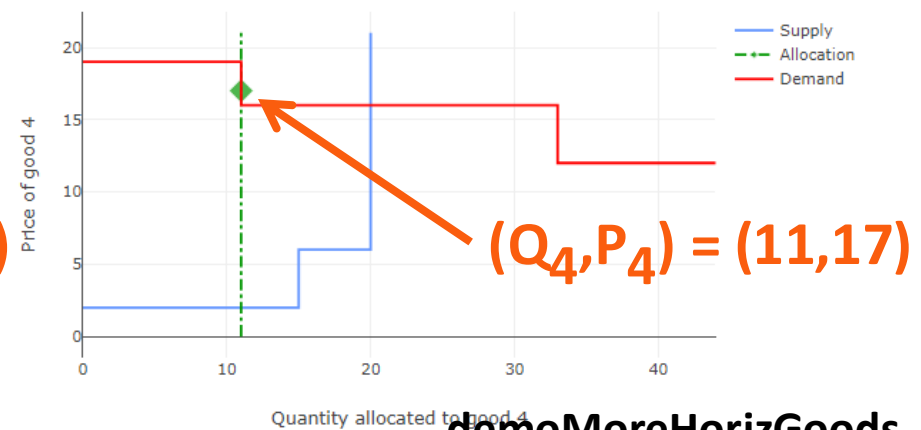
Supply curve and unadjusted demand\* for good 2

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



Supply curve and unadjusted demand\* for good 4

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



## 6. More Than Two Varieties

### Allocations of bids to goods



## 6. More Than Two Varieties Bidders/Bids

Various bids for different quantities,  
specifying prices for up to 3 goods

Bidder 1

Bid	Units	Price	Price	Price
1	3 <b>3</b>	17 <b>17</b>	10 <b>10</b>	9 <b>9</b>

Bidder 5

Bid	Units	Price	Price	Price
1	2 <b>2</b>	8 <b>8</b>	2 <b>2</b>	9 <b>9</b>

Bidder 2

Bid	Units	Price	Price	Price
1	6 <b>6</b>	0 <b>0</b>	12 <b>12</b>	8 <b>8</b>

Bidder 6

Bid	Units	Price	Price	Price
1	5 <b>5</b>	5 <b>5</b>	8 <b>8</b>	7 <b>7</b>

Bidder 3

Bid	Units	Price	Price	Price
1	4 <b>4</b>	7 <b>7</b>	14 <b>14</b>	8 <b>8</b>

Bidder 7

Bid	Units	Price	Price	Price
1	4 <b>4</b>	10 <b>10</b>	8 <b>8</b>	12 <b>12</b>

Bidder 4

Bid	Units	Price	Price	Price
1	9 <b>9</b>	16 <b>16</b>	12 <b>12</b>	11 <b>11</b>

Bidder 8

Bid	Units	Price	Price	Price
1	7 <b>7</b>	4 <b>4</b>	4 <b>4</b>	5 <b>5</b>

## 6. Three Varieties Supply Curves

Supply curve for good 1

Units	Price
<input type="text" value="10"/> 10	<input type="text" value="0"/> 0

Supply curve for good 2

Units	Price
<input type="text" value="10"/> 10	<input type="text" value="0"/> 0

Supply curve for good 3

Units	Price	
<input type="text" value="5"/> 5	<input type="text" value="0"/> 0	<input type="button" value="x"/>
<input type="text" value="5"/> 5	<input type="text" value="4"/> 4	<input type="button" value="x"/>

TQSS

TQSS enabled

Enabled

Price normalisation and scaling

Normalised - constrain supply (may constrain quantities below supply curves specif

Units	Price
<input type="text" value="60"/> 60	<input type="text" value="0"/> 0

# 6. Three Varieties

## Demand, Supply & Allocations

Supply curve for good 1   Supply curve

Units	Price
10	0

Supply curve for good 2   Supply curve

Units	Price
10	0

Supply curve for good 3   Supply curve

Units	Price	
5	0	<input type="button" value="x"/>
5	4	<input type="button" value="x"/>

TQSS

TQSS enabled

Enabled

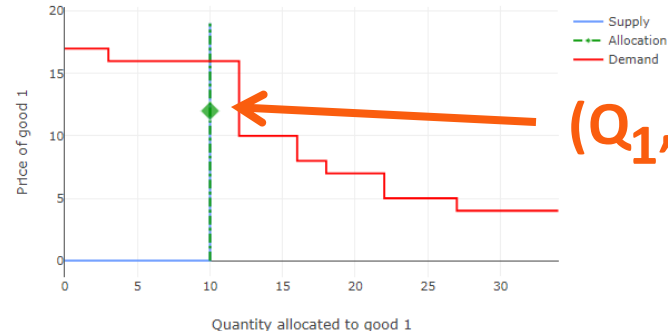
Price normalisation and scaling

Normalised - constrain supply (may constrain quantities below supply curves specif

Units	Price
60	0

Supply curve and unadjusted demand\* for good 1

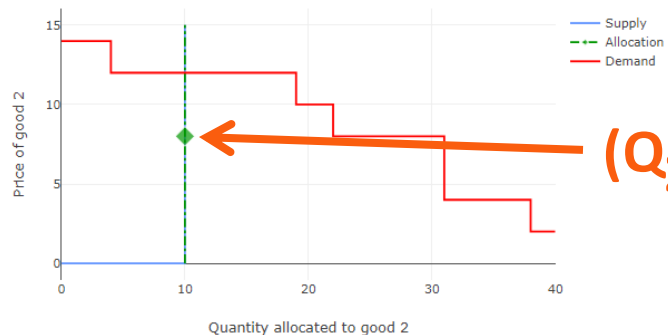
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$$(Q_1, P_1) = (10, 12)$$

Supply curve and unadjusted demand\* for good 2

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$$(Q_2, P_2) = (10, 8)$$

Supply curve and unadjusted demand\* for good 3

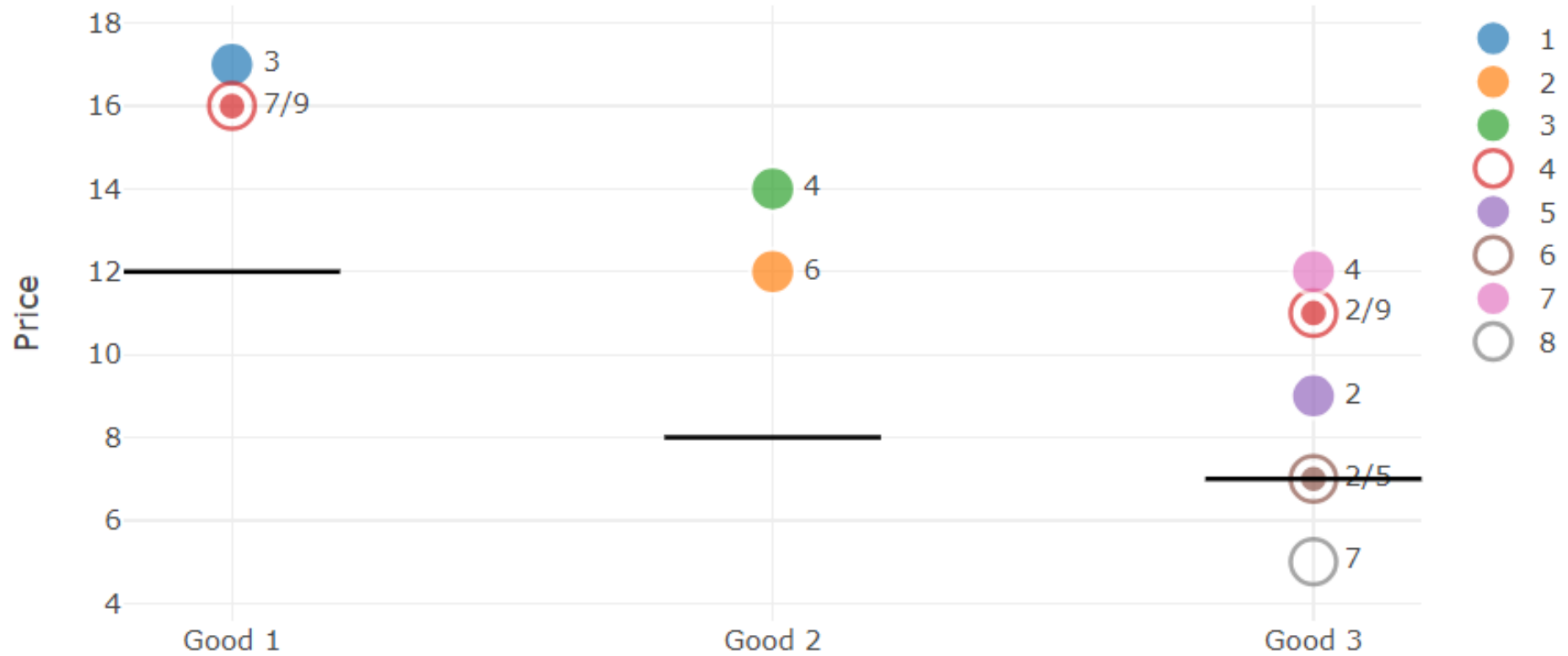
\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



$$(Q_3, P_3) = (10, 7)$$

## 6. More Than Two Varieties

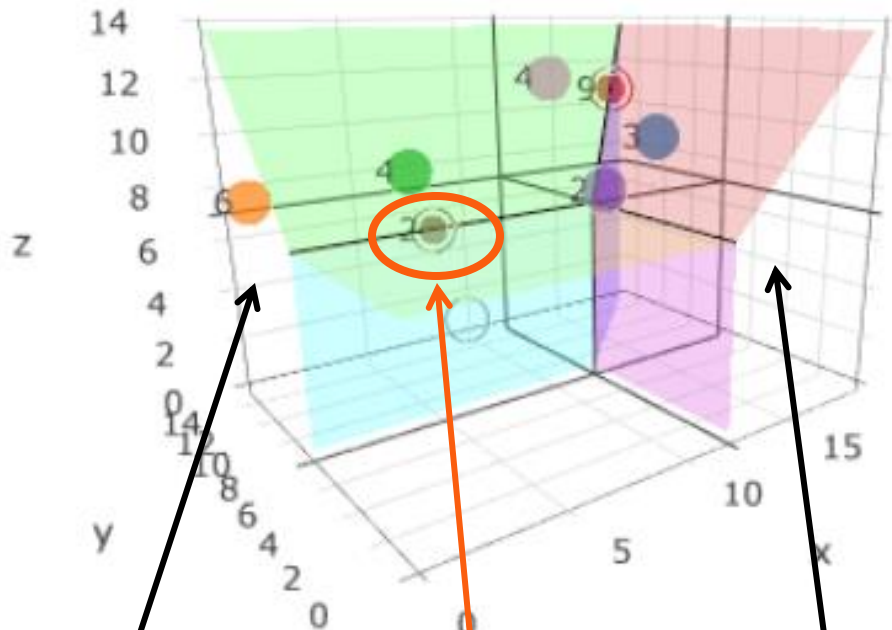
### Allocations of bids to goods



## 6. More Than Two Varieties

### Bids in price space

*Two different views of the same 3D-graph*



Bids

Bid indifferent

Bids

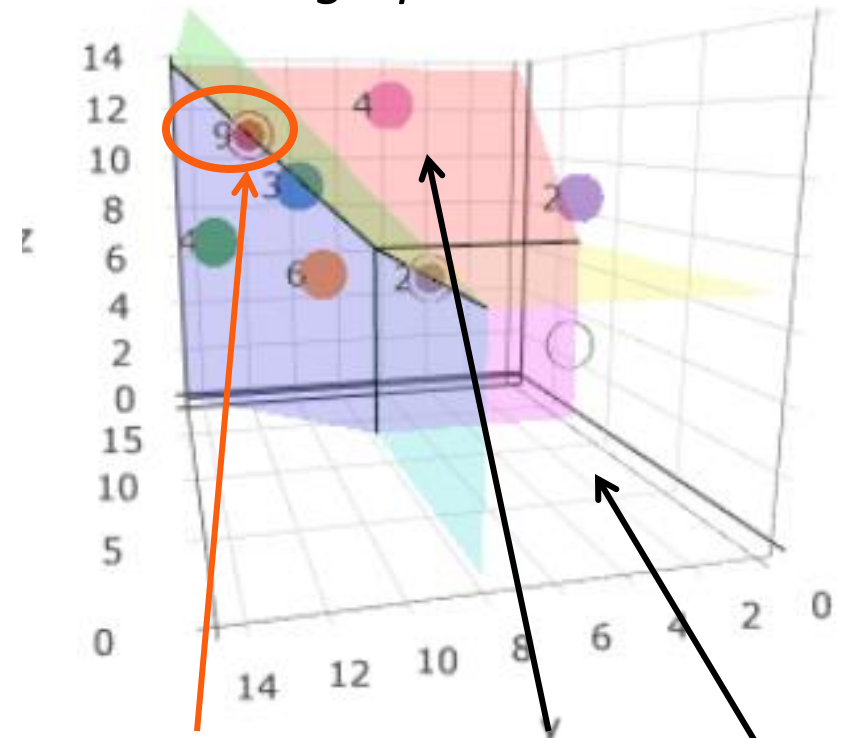
allocated  
to good 2,  
e.g. incl.

between good 2  
and good 3  
and no purchase

allocated  
to good 1,  
e.g. incl.

● 6 ● 4

● 3



Bid indifferent

Bids

allocated  
to good 2,  
e.g. incl.

between good 2  
and good 3

allocated  
to good 3,  
e.g. incl.

No sale

● 4 ● 2

# Section 7

## Section 7.2



## 7.2 General Trade-offs and Additional Constraints Bidders/Bids

Overall bid quantity constraint  $4 \times (\text{Quantity good 1}) + 5 \times (\text{Quantity good 2}) \leq 10$

Good 1 constraint  $4 \times (\text{Quantity good 1}) \leq 8$

Good 2 constraint  $5 \times (\text{Quantity good 2}) \leq 6$

Overall bid  
quantity

Bidder 1

Overall bid quantity  
 10

Bid values

Good	Trade-off	Units	Price	
<input type="text" value="1"/>	<input type="text" value="4"/> 4	<input type="text" value="8"/> 8	<input type="text" value="5"/> 5	<input type="button" value="x"/>
<input type="text" value="2"/>	<input type="text" value="5"/> 5	<input type="text" value="6"/> 6	<input type="text" value="8"/> 8	<input type="button" value="x"/>

Good 1

Good 2

Trade-off  
good 1 : good 2

Maximum  
quantities for  
good 1 / good 2

Bid prices for  
good 1 / good 2

[Generalised  
paired bid  
with 4:5  
trade-off]

## 7.2 General Trade-offs and Additional Constraints Bidders/Bids

Overall bid quantity constraint  $2 \times (\text{Quantity good 1}) + 1 \times (\text{Quantity good 2}) \leq 10$

Good 1 constraint  $2 \times (\text{Quantity good 1}) \leq 10$

Good 2 constraint  $1 \times (\text{Quantity good 2}) \leq 10$

Overall bid  
quantity

Bidder 2

Overall bid quantity  
10 10

Bid values

Good	Trade-off	Units	Price
1	2	10	7
2	1	10	5

+ Bid value

Good 1

Good 2

Trade-off  
good 1 : good 2

Maximum  
quantities for  
good 1 / good 2  
(= overall bid quantity)

Bid prices for  
good 1 / good 2

[Generalised  
paired bid  
with 2:1  
trade-off]

## 7.2 General Trade-offs and Additional Constraints Bidders/Bids

Overall bid quantity constraint  $1 \times (\text{Quantity good 1}) \leq 30$

Good 1 constraint  $1 \times (\text{Quantity good 1}) \leq 30$

Overall bid  
quantity

Bidder 3  Bidder

Bid 1  Bid

Overall bid quantity  30

Bid values

Good	Trade-off	Units	Price
1	1	30	4

Good 1

Maximum  
quantity

Bid price

[Standard  
bid for  
good 1 only]

Overall bid quantity constraint  $1 \times (\text{Quantity good 2}) \leq 30$

Good 2 constraint  $1 \times (\text{Quantity good 2}) \leq 30$

Overall bid  
quantity

Bid 2  Bid

Overall bid quantity  30

Bid values

Good	Trade-off	Units	Price
2	1	30	4

Good 2

Maximum  
quantity

Bid price

[Standard  
bid for  
good 2 only]

demoGenTradeoffs

## 7.2 General Trade-offs and Additional Constraints Supply Curves

Supply

Supply Ordering

HorizontalSupply

Supply curves for each good

Supply curve for good 1

Units	Price
<input type="text" value="30"/> 30	<input type="text" value="0"/> 0

Supply curve for good 2

Units	Price
<input type="text" value="30"/> 30	<input type="text" value="0"/> 0

# 7.2 General Trade-offs and Additional Constraints

## Demand, Supply & Allocations

Supply

Supply Ordering

HorizontalSupply

Supply curves for each good

Supply curve for good 1   Supply curve

Units	Price
<input type="text" value="30"/> 30	<input type="text" value="0"/> 0

Supply step

Supply curve for good 2   Supply curve

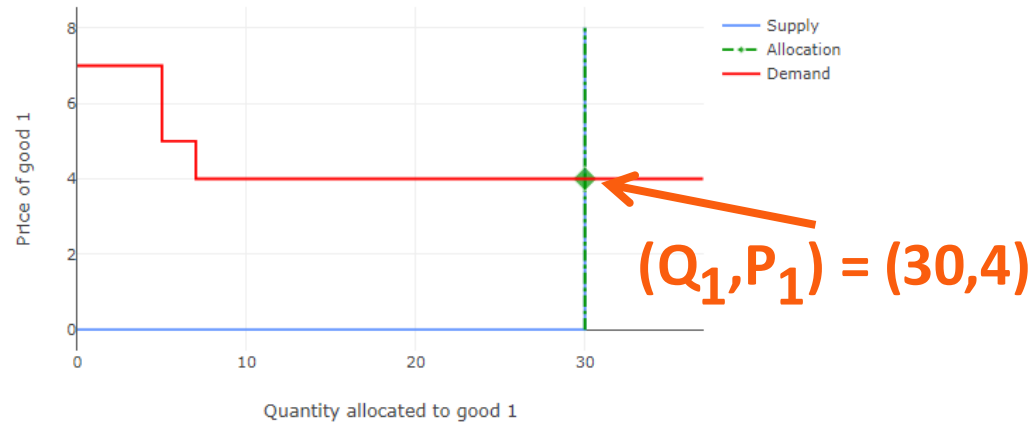
Units	Price
<input type="text" value="30"/> 30	<input type="text" value="0"/> 0

Supply step

Supply curve

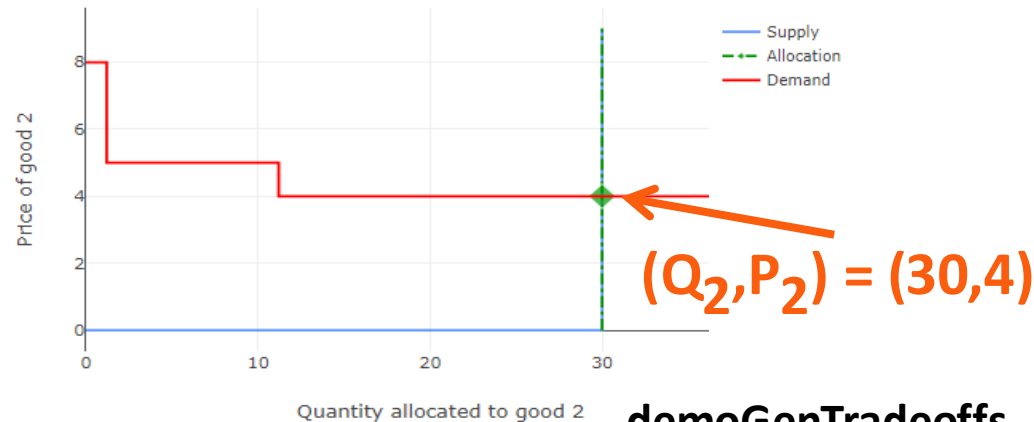
Supply curve and unadjusted demand\* for good 1

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



Supply curve and unadjusted demand\* for good 2

\*Supply curve ignores TQSS, and unadjusted demand ignores all bidding on other goods



## 7.2 General Trade-offs and Additional Constraints

### Prices & Allocation of Bids to Goods

$$4 \times (Q_1 = 1) + 5 \times (Q_2 = 1.2) = 10$$

*Bidder 1's allocation + constraints*

$$4 \times (Q_1 = 1) \leq 8$$

$$5 \times (Q_2 = 1.2) = 6$$

Auction output

Prices and total allocations

	Good 1		Good 2	
Price	4	4	4	4
Allocation	30	30	30	30

Allocations

Bidder	Good 1		Good 2	
1	1	1	1.2	1.2
2	5	5	0	0
3	24	24	28.8	28.8

## 7.2 General Trade-offs and Additional Constraints

### Prices & Allocation of Bids to Goods

$$2 \times (Q_1 = 5) + 1 \times (Q_2 = 0) = 10$$

*Bidder 2's allocation + constraints*

$$2 \times (Q_1 = 5) = 10$$

$$1 \times (Q_2 = 0) = 10$$

Auction output

Prices and total allocations

	Good 1		Good 2	
Price	4	4	4	4
Allocation	30	30	30	30

Allocations

Bidder	Good 1		Good 2	
1	1	1	1.2	1.2
2	5	5	0	0
3	24	24	28.8	28.8

## 7.2 General Trade-offs and Additional Constraints

### Prices & Allocation of Bids to Goods

$$1 \times (Q_1 = 24) \leq 30$$

$$1 \times (Q_1 = 24) \leq 30$$

*Bidder 3's allocation + constraints*

$$1 \times (Q_2 = 28.8) \leq 30$$

$$1 \times (Q_2 = 28.8) \leq 30$$

Auction output

Prices and total allocations

	Good 1		Good 2	
Price	4	4	4	4
Allocation	30	30	30	30

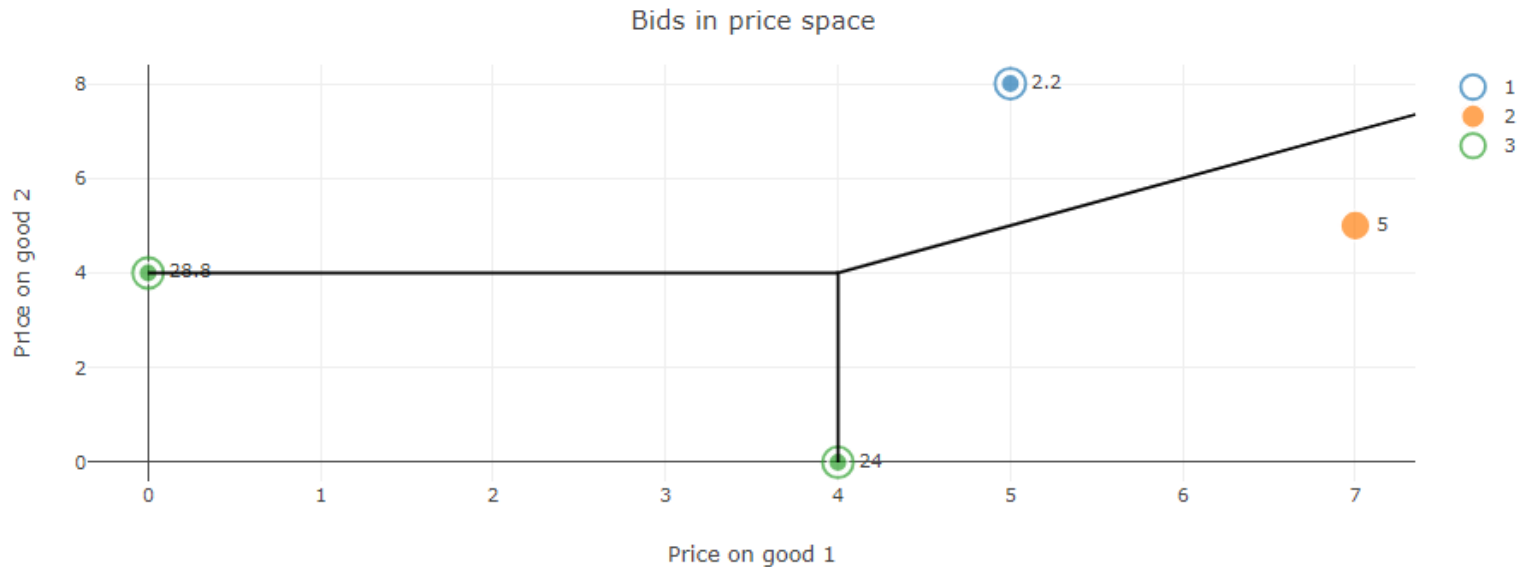
Allocations

Bidder	Good 1		Good 2	
1	1	1	1.2	1.2
2	5	5	0	0
3	24	24	28.8	28.8



## 7.2 General Trade-offs and Additional Constraints

### Bids in price space & Allocations of bids to goods



## Section 7.3

## 7.3 Profit Maximisation Bidders/Bids

Bidder 1 ▼ ✕ Bidder

Bid	Units	Price	
1	<input type="text" value="4"/> 4	<input type="text" value="17"/> 17	<span>✕</span>
2	<input type="text" value="8"/> 8	<input type="text" value="11"/> 11	<span>✕</span>
3	<input type="text" value="5"/> 5	<input type="text" value="7"/> 7	<span>✕</span>

+ Bid

Bidder 2 ▼ ✕ Bidder

Bid	Units	Price
1	<input type="text" value="11"/> 11	<input type="text" value="9"/> 9

+ Bid

Bidder 3 ▼ ✕ Bidder

Bid	Units	Price
1	<input type="text" value="11"/> 11	<input type="text" value="8"/> 8

+ Bid

Bidder 4 ▼ ✕ Bidder

Bid	Units	Price
1	<input type="text" value="11"/> 11	<input type="text" value="6"/> 6

+ Bid

## 7.3 Profit Maximisation Supply Curves

Supply

Supply Ordering

HorizontalSupply

Supply curves for each good

Supply curve for good 1

Units	Price
<input type="text" value="30"/>	<input type="text" value="0"/>

+ Supply step

+ Supply curve for good

TQSS

TQSS enabled

Disabled

Maximisation strategy

Maximise profitability  
[was Maximise efficiency]

# 7.3 Profit Maximisation

## Prices & Allocation of Bids to Goods

Auction output  
**Auctioneer's profit** 240

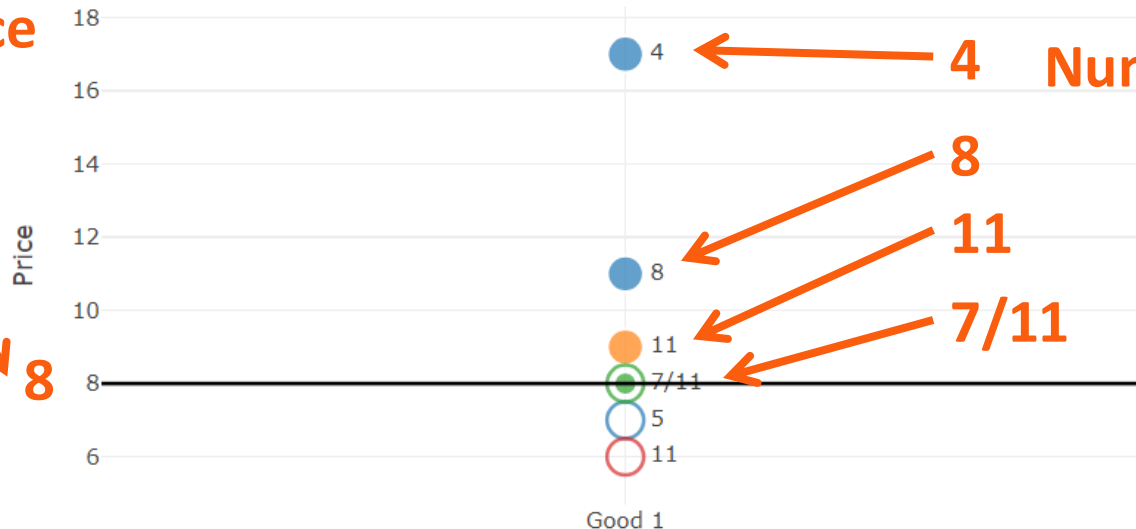
Prices and total allocations

	Good 1
Price	8
Allocation	30

Allocations

Bidder	Good 1
1	12
2	11
3	7

**Uniform Price**



**4** Number of units allocated

**8**

**11**

**7/11**

## 7.3 Profit Maximisation Supply Curves

Supply

Supply Ordering

HorizontalSupply

Supply curves for each good

Supply curve for good 1

Units	Price
<input type="text" value="30"/> 30	<input type="text" value="5"/> 5

+ Supply step

+ Supply curve for good

TQSS

TQSS enabled

Disabled

Maximisation strategy

Maximise profitability  
[was Maximise efficiency]

# 7.3 Profit Maximisation

## Prices & Allocation of Bids to Goods

**Auctioneer's profit** Auction output  
 Auctioneer's profit: 92 ( $92 = 23 \times (9 - 5) > 30 \times (8 - 5) = 90$ )

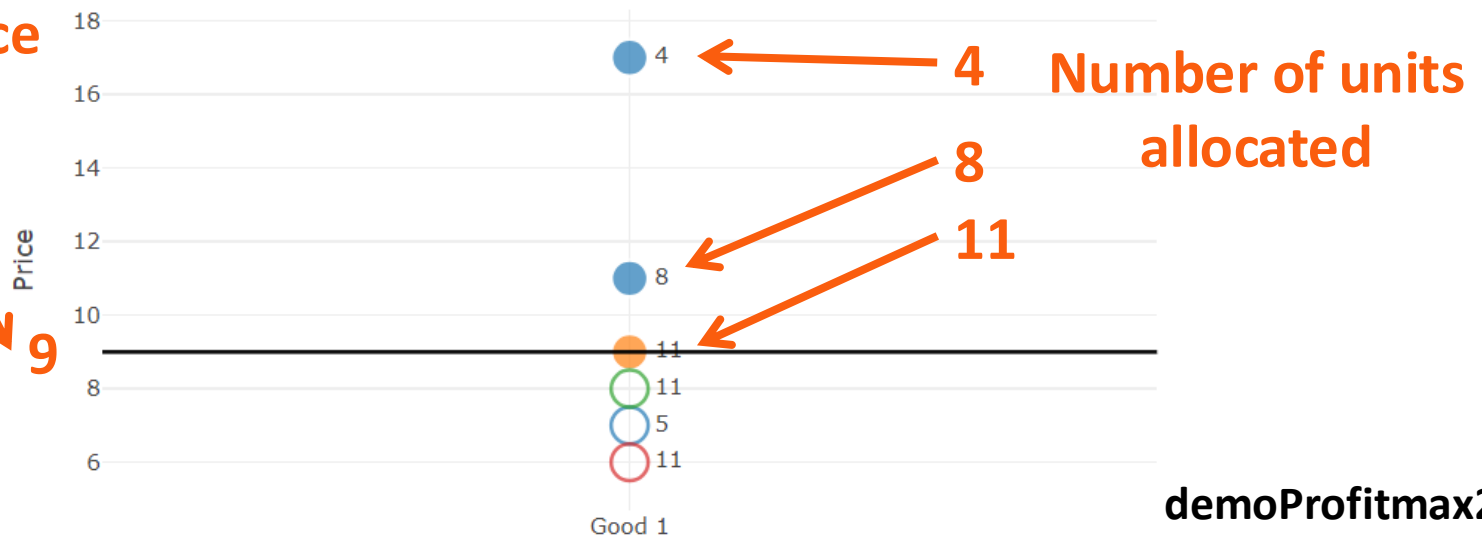
Prices and total allocations

	Good 1	
Price	9	9
Allocation	23	23

Allocations

Bidder	Good 1
1	12
2	11

**Uniform Price**



## 7.3 Profit Maximisation Supply Curves

Supply

Supply Ordering

HorizontalSupply

Supply curves for each good

Supply curve for good 1

Units	Price
<input type="text" value="30"/> 30	<input type="text" value="7"/> 7

+ Supply step

+ Supply curve for good

TQSS

TQSS enabled

Disabled

Maximisation strategy

Maximise profitability  
[was Maximise efficiency]



# 7.3 Profit Maximisation

## Prices & Allocation of Bids to Goods

Auction output  
**Auctioneer's profit** 48 ( $= 12 \times (11 - 7) > 23 \times (9 - 7) = 46$ )

Prices and total allocations ▼

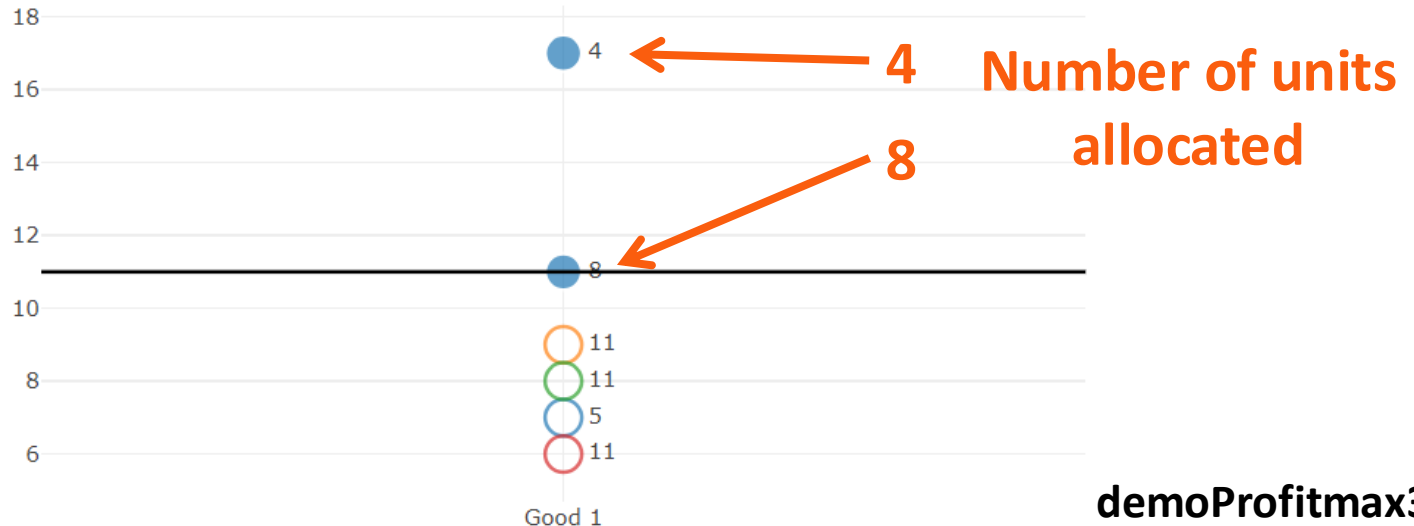
	Good 1
Price	11
Allocation	12

Allocations ▼

Bidder	Good 1
1	12

**Uniform Price**

11



## Section 7.4

## 7.4 Budget-Constrained Version: Iceland's Auction Bidders/Bids

Bids

Bid	Budget	Price	Price	
1	7 7	1 1	6 6	×
2	6 6	2 2	7 7	×
3	3 3	3 3	0 0	×
4	3 3	1 1	5 5	×

+ Bid

Budget: Maximum  
total spend on  
good 1 or good 2

Maximum  
unit price  
good 1

Maximum  
unit price  
good 2

## 7.4 Budget-Constrained Version: Iceland's Auction Supply Curves

Supply curves for each good

Supply curve for good 1

Units	Price
<input type="text" value="10"/> 10	<input type="text" value="0"/> 0

Supply curve for good 2

Units	Price
<input type="text" value="10"/> 10	<input type="text" value="2"/> 2

## 7.4 Budget-Constrained Version: Iceland's Auction Prices & Allocation of Bids to Goods

Auction output

Auctioneer's profit: 15.4 **15.4 Auctioneer's profit**

Prices and total allocations

	Good 1		Good 2	
Price	1	<b>1</b>	5	<b>5</b>
Allocation	10	<b>10</b>	1.8	<b>1.8</b>

Allocations

Bid	Good 1		Good 2	
1	0		1.4	
2	6	<b>6</b>	0	<b>0</b>
3	3		0	
4	1		0.4	

**Bidder 2's surplus on good 1:  $6/1 \times (2-1) = 6$**

**Bidder 2's surplus on good 2:  $6/5 \times (7-5) = 2.4$**

**i.e. allocated only good 1**